ECE 301 (Section 001) Homework 8
Spring 2025, Dr. Chau-Wai Wong
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Problem 1 (Deep Learning with Matlab) In recent updates, Matlab has put together well-guided tutorials for deep learning. This is one set of tutorials on deep-learning-based "Image Data Workflows".

a) Complete one of the tutorials on the provided web page by running their example code. Write a concise report consisting of key source code, images, and your explanations.

b) (Bonus, 10 pts) Complete tutorial: "Create Simple Deep Learning Network for Classification." Write a concise report consisting of key source code, images, and your explanations.

c) (Bonus, 10 pts) Complete tutorial: "Prepare Network for Transfer Learning Using Deep Network Designer." Write a concise report consisting of key source code, images, and your explanations.

These tutorials may give you ideas about your projects in other courses.

Problem 2 (Projection and PCA) You are given a data matrix

$$\mathbf{X} = \begin{bmatrix} -0.92 & 1.09 & -1.35 & 2.06 & -0.60 & -0.28 \\ -0.08 & 1.15 & -1.67 & 1.08 & -1.14 & 0.66 \end{bmatrix}.$$

and its principal component $\mathbf{u} = [0.7474,\ 0.6644]^T$.

a) Plot the data points using circles and the principal component using a line segment with one end at $[0, 0]^T$ on a 2D plane using Matlab.

b) Calculate the projection c_i for each data point $\mathbf{x}_i$ to $\mathbf{u}$. Note that c_i can be negative.

c) Calculate the sample variance of $\{c_i\}_{i=1}^n$, namely,

$$\widehat{\mathrm{Var}}\left(\{c_i\}\right) = \frac{1}{n-1}\sum_{i=1}^{n}\left(c_i - \bar{c}\right)^2,$$

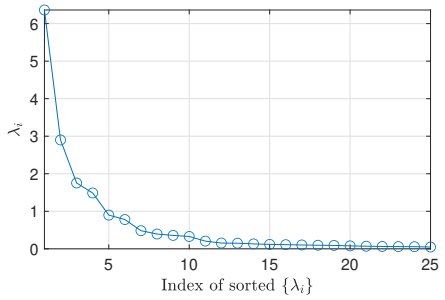
where $\bar{c} \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} c_i$ is the sample mean.

d) Repeat b) and c) using another unit vector $\mathbf{u}_a = [0.8,\ 0.6]^T$. Is the newer sample variance smaller than that in b)? Can you explain why?

Problem 3 (PCA on Downsampled Yale Face Database) In this problem, we will explore PCA as a visualization tool for Yale Face Database. Download the `.m` files and the database. Extract the face image files into a folder named `yalefaces` and put the `.m` files at the same level of the folder. Call this folder "problem_pca_face/". Open Matlab, change your "Current Folder" to "problem_pca_face/", and open `main_pca_visualization.m`.

a) Run the code corresponding to this part only, describe the data structure of variable `img_buffer`. Set `preview_img_flag` to `1`, re-run the code to visually inspect the whole database.

b) Complete Matlab function `[V, Lambda_mat] = PcaViaKlt(data)` by implementing PCA using eigendecomposition on a sample covariance matrix of the face data. The detailed information about the input and outputs are given in the comments of the incomplete function. You may use built-in function `eig` for eigendecomposition. If your implementation is correct, after running the code of b), you will obtain a plot similar to the following.



c) Run the code of c) to visualize a few dominating eigenvectors. Comment on whether they reflect some characteristics of the faces you saw in a).

d) The code of d) projects each face image (coming from one of the four selected classes) onto a 2D space. Comment on PCA's data visualization performance in this specific example.

Problem 4 (Orthogonal Projection) Consider the set of inconsistent linear equations $\mathbf{Ax} = \mathbf{b}$ given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

a) Find the least-squares solution to these equations.

b) Find the "hat" matrix $\mathbf{H}$. Using Matlab, numerically verify $\mathbf{H} = \mathbf{H}^2$. Argue why.

c) Find the best approximation $\hat{\mathbf{b}} = \mathbf{Hb}$ to $\mathbf{b}$. Find the vector $\mathbf{b}' = (\mathbf{I} - \mathbf{H})\mathbf{b}$ and show numerically that it is orthogonal to $\hat{\mathbf{b}}$.

d) What does the matrix $\mathbf{I} - \mathbf{H}$ represent? If $\mathbf{H}$ is called the "orthogonal projector," can you think of a name for $\mathbf{I} - \mathbf{H}$? Numerically verify $\mathbf{I} - \mathbf{H} = (\mathbf{I} - \mathbf{H})^2$. Argue why.

e) In a 3-dimensional coordinate system, draw the column vectors of matrix $\mathbf{A}$, the column vector space of $\mathbf{A}$, $\mathbf{b}$, $\hat{\mathbf{b}}$, and $\mathbf{b}'$. Make sure that the drawing is reasonably accurate which can reflect the relationship among these quantities.

Problem 5 (Mid-Semester Class Evaluation, Bonus 10') To get the bonus points, please take a screenshot of the confirmation page after submitting a survey and use the confirmation screenshot as the answer to this problem.

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.