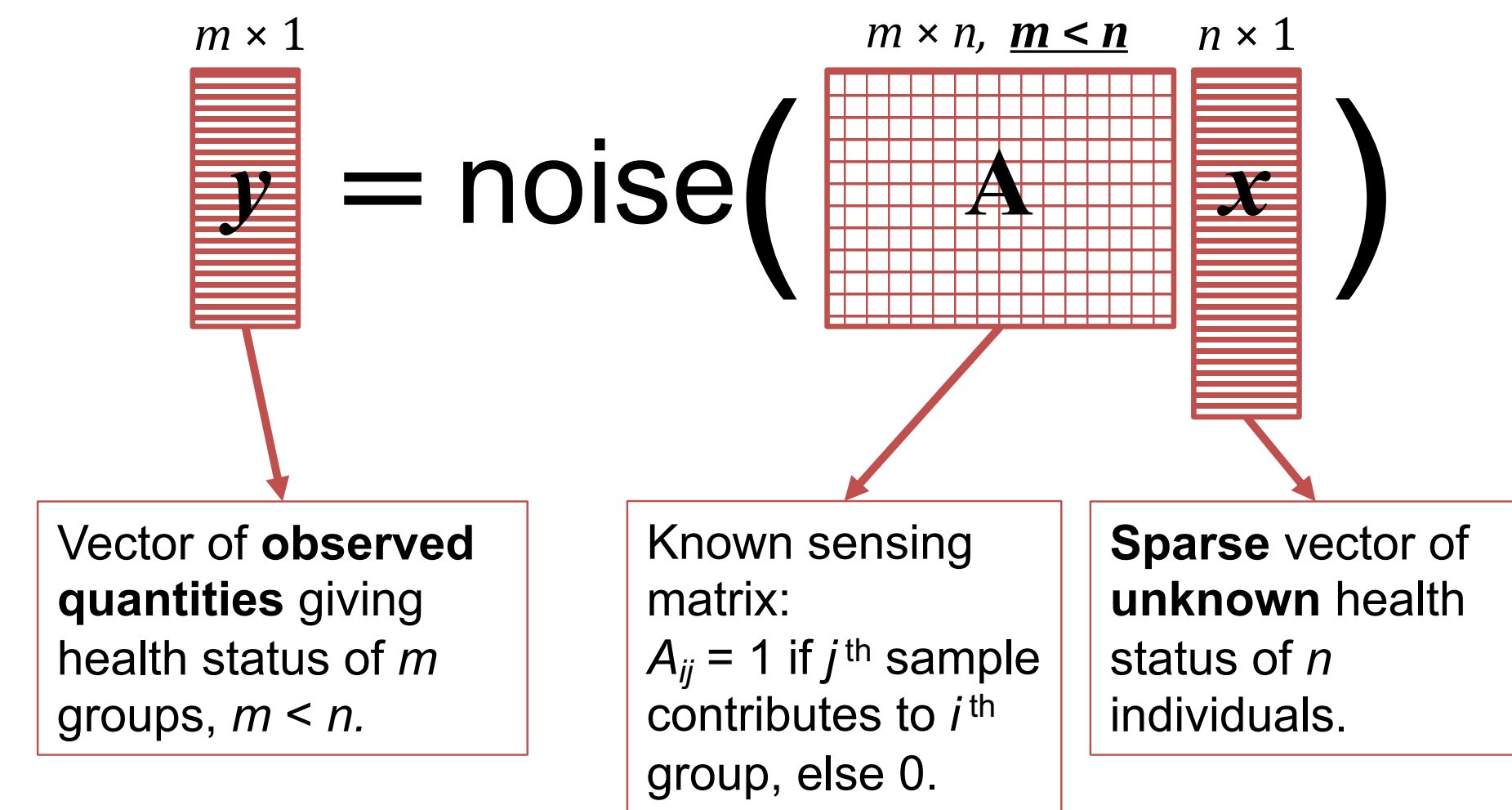


## Group Testing: Motivation & Compressed Sensing Formulation

- Need widespread testing for COVID-19 – pooled or **group testing** for improving efficiency.
- Initial idea by Robert Dorfman in 1943 – divide individuals into groups of fixed size; if a group tests negative, all individuals in the group are declared healthy, else proceed to individual testing.
- **Compressed sensing** approach to group testing:
  - Mix  $n$  individual samples into  $m < n$  pools.
  - $\mathbf{y} = \mathcal{N}(\mathbf{Ax})$ ,  $\mathcal{N}$  is a stochastic noise model.
  - Estimate  $\mathbf{x}$  given  $\mathbf{y}$ ,  $\mathbf{A}$  and the noise model.
  - Use of different decoding algorithms.
  - Single stage (**nonadaptive**), unlike Dorfman pooling, which is more time consuming.
  - Can incorporate **side information** (SI) into the probability distribution of  $\mathbf{x}$ .



## Research Question

- Can group testing efficiency be improved by utilizing side information (SI) in the following forms?
- Family structure: nonoverlapping or overlapping.
  - Contact tracing data: (i) who's in contact, (ii) physical proximity, and (iii) contact duration.

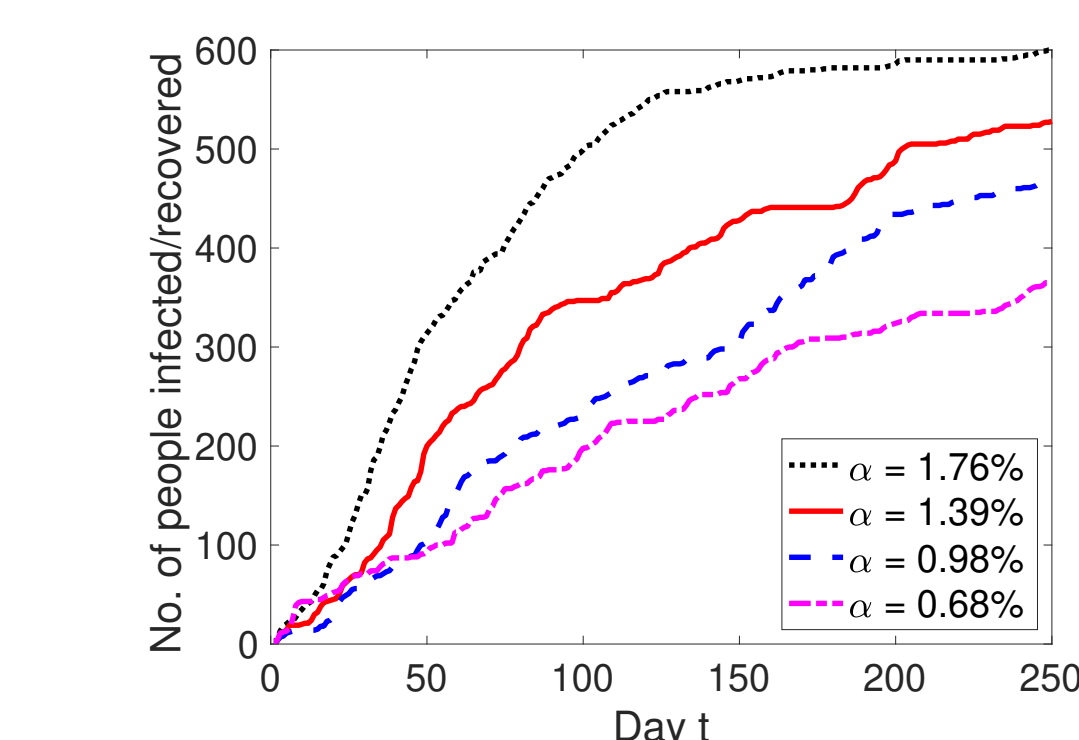
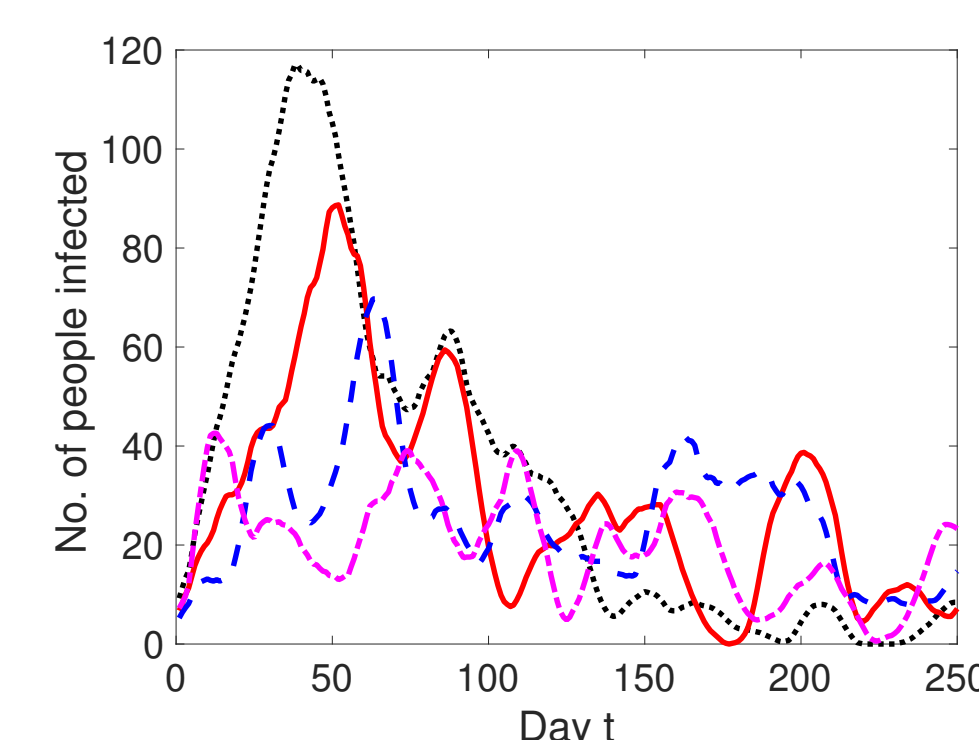
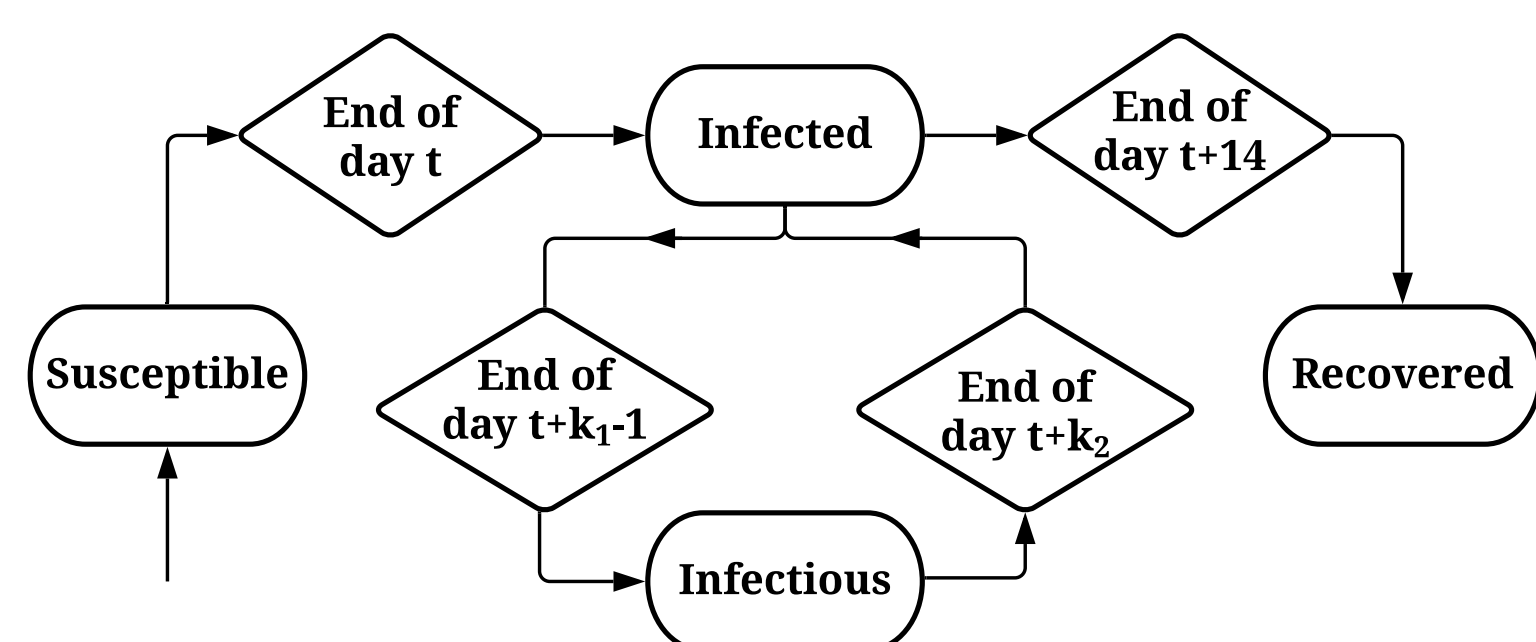
## Recent Progress

- Compressed sensing formulation solved using a message-passing style algorithm [1].
- Multiplicative noise model with an end-to-end study of decoding in pooled quantitative PCR [2].
- Exploited community structure with a focus on encoder design, but did not consider noise [3, 4].

[1] Zhu, Rivera, and Baron, "Noisy pooled PCR for virus testing," Apr. 2020.  
 [2] Ghosh et al., "A compressed sensing approach to group testing for COVID-19 detection," May 2020.  
 [3] Nikolopoulos, Guo, Fragouli, and Diggavi, "Community aware group testing," Jul. 2020.  
 [4] Nikolopoulos, Srinivasavaradhan, Guo, Fragouli, and Diggavi, "Group testing for overlapping communities," Dec. 2020.

## Infection Model

- Use SEIR style dynamical infection model incorporating contact tracing SI for data simulation.



(a) State transition diagram for an individual (b) Number of active infections (c) Cumulative number of infections

## Proposed Methods

### Binary Noise: Model M1

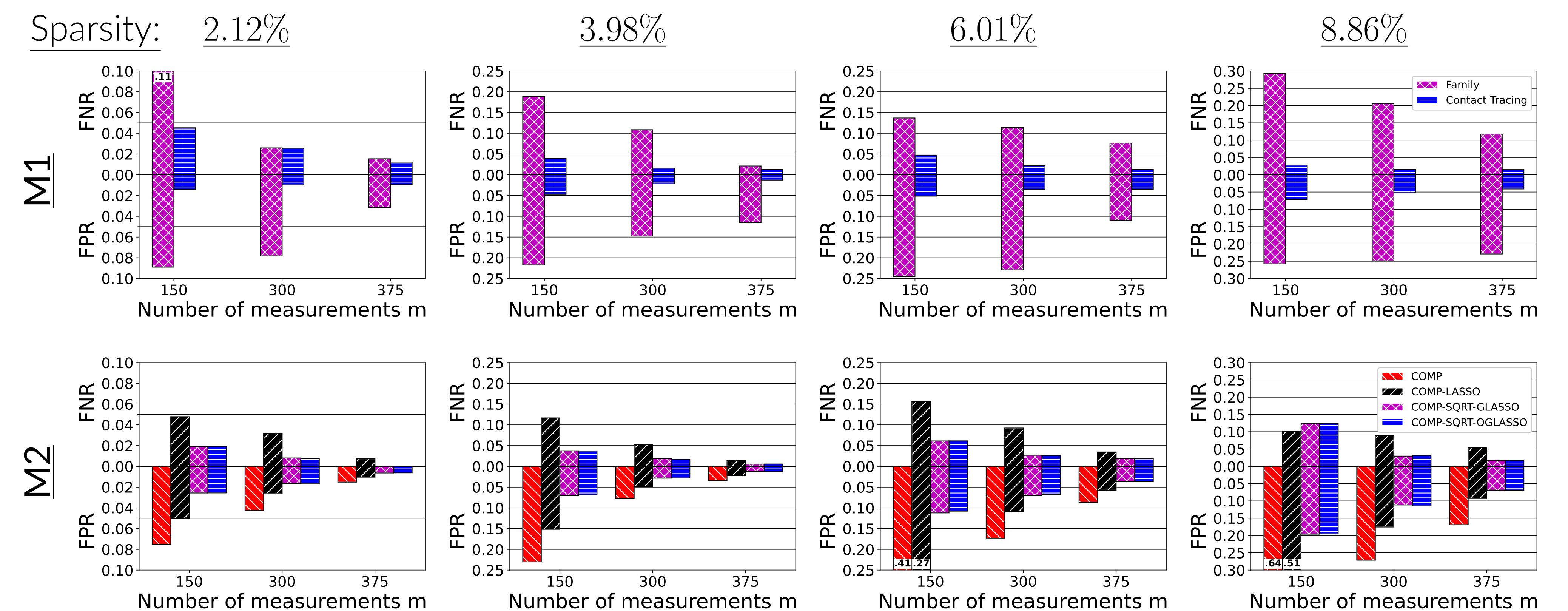
- Pooling:  $\mathbf{w} = \mathbf{Ax}$ , where  $\mathbf{A} \in \{0, 1\}^{m \times n}$  and  $\mathbf{x} \in \{0, 1\}^n$ .
- Noise:  $y_i \in \{0, 1\}$  with  $\Pr(y_i = 1 | w_i = 0) = 0.1\%$  and  $\Pr(y_i = 0 | w_i > 0) = 2\%$ .
- Decoding: **Generalized Approximate Message Passing** (GAMP) framework.
  - The key is to design a denoiser  $g_{\text{in}}$  with  $\hat{x}_i = g_{\text{in}}(\mathbf{v}) = \mathbb{E}[X_i | \mathbf{V} = \mathbf{v}]$ , where  $\mathbf{v}$  is pseudo data.
  - Design two denoisers – family denoiser and contact tracing denoiser.

### Multiplicative Noise: Model M2

- Pooling:  $\mathbf{w} = \mathbf{Ax}$ , where  $\mathbf{A} \in \{0, 1\}^{m \times n}$  and  $\mathbf{x} \in [0, \infty)^n$ .
- Noise:  $\mathbf{y} = \mathbf{w} \circ (1 + q)^{\mathcal{N}(0, \sigma^2 \mathbf{I})}$ , where  $q \in (0, 1]$  and  $\sigma^2 \ll 1$  are known parameters.
- Decoding: Least Absolute Shrinkage and Selection Operator (LASSO) based algorithms.
  - COMP followed by group square-root LASSO for family structure.
  - COMP followed by **overlapping group square-root LASSO** for contact tracing structure.

## Experimental Results

- Figure below shows performance of methods M1 and M2 for a population of  $n = 1000$  individuals.
- M1: FNR & FPR < 5% for up to 6% sparsity; contact tracing data helps more than family structure.
- M2: Significant improvement using SI in decoding; allows estimation of individual viral loads.



## Conclusion and Future Directions

- Improved efficiency by exploiting family structure and contact tracing data – achieved  $\sim 5\%$  FNR & FPR at 4% prevalence with need for only 15% of tests required in a conventional testing scenario.
- Could incorporate SI into encoder design and generate better sensing matrices with flexible sizes.