

ECE 301 (Section 001) Homework 11, Spring 2021

Problem 1 (DTFT and Inverse DTFT) Solve the following using the definitions *or* the properties.

- a) $\mathcal{F}\{u[n-2] - u[n-6]\}$
- b) $\mathcal{F}\{2^n \sin(\pi n/4)u[-n]\}$
- c) $\mathcal{F}\{\sin(\pi n/2) + \cos(7\pi n/3)\}$
- d) $\mathcal{F}^{-1}\left\{\frac{e^{-j\omega} - \frac{1}{3}}{1 - \frac{1}{3}e^{-j\omega}}\right\}$

Problem 2 (Triangle, Rectangle, and Sinc Functions)

- a) Using the definition of convolution, show via integration that $y(t) = H \text{rect}(t/2W) * H \text{rect}(t/2W)$ is a triangle. What are the width and height of the triangle?
- b) Use the definition or the properties to compute the inverse DTFT of the following frequency functions $X(e^{j\Omega})$.

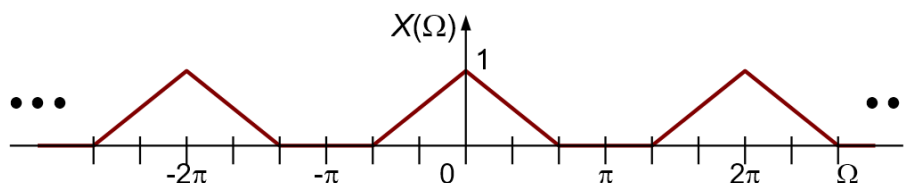


Figure 1: Signal for inverse DTFT calculation.

Problem 3 (10', bonus) (Windowing Effect and Frequency Resolution) In this problem, we will investigate the frequency resolution of Fourier transform. We investigate two neighboring musical notes, C_4 at $f_1 = 261.63$ Hz and $C_4^\#$ at $f_2 = 277.18$ Hz. You can play with a virtual piano here:

<https://recursivearts.com/virtual-piano/>

We make an oversimplified assumption that the audio signal of each musical note is in the form of $x_i(t) = \cos(2\pi f_i t)$, $t \in \mathbb{R}$. A causal time window $t \in [0, 2T]$ of unit gain is applied to “generate” signals of finite length. (To simplify the problem, we investigate the continuous-time signal. The analysis of the discrete-time signal is similar.)

- a) With the help of the CTFT table, calculate the Fourier spectra for the windowed $x_i(t)$, for $i = 1, 2$.
- b) What is the angular frequency of the first zero-crossing on the right-hand side of the magnitude spectrum of note C_4 ? What is the angular frequency of the first zero-crossing on the left-hand side of the magnitude spectrum of note $C_4^\#$?

- c) What is the smallest T that can clearly separate the peaks of the two neighboring musical notes?

Group Study (1', bonus) Take a screenshot of the whole team with everyone's camera capturing his/her face. One of you will share a window showing the specific homework assignment sheet that you are working on. Include the screenshot in your own homework submission as Problem 4. Your screenshot gets you 1 bonus point; your group members need to do it separately to earn theirs.