ECE 301 (Section 001) Homework 12, Spring 2021
(Due on Friday, 4/23 at 4:30 pm)

Problem 1 (DFT Calculations and Convolution Property)

a) (6') Compute the DFT of the following signals by hand and plot $X[k]$. Label the low and high frequencies. Feel free to verify your results using Matlab command `fft`. For the following problems, we use vector notation to represent finite length signals. For example $\mathbf{x} = [-1, 2, 3]$ means a signal with $x[0] = -1$, $x[1] = 2$, and $x[2] = 3$.

(a) DFT of $[1, 1, -1, -1]$.

(b) DFT of $[1, 0, 1, 0]$.

(c) Inverse DFT of $[0, 0, 1, 0]$.

b) Quiz 3 (Linear Convolution via Circular Convolution) Follow-up (4')

(i) Please attach the screenshot of your Quiz 3 grade.

(ii) If your score is less than 4' or 100% (including those who missed the quiz), you may recover at most 2' by completing the following question.

Show that the linear convolution result of $\mathbf{x} * \mathbf{y}$, where $\mathbf{x} = [1, 1]$ and $\mathbf{y} = [1, 1, -1, -1]$ is the same as the result of $\mathrm{IDFT}\left\{\mathrm{DFT}([\mathbf{x}, 0, \ldots, 0])\,\mathrm{DFT}([\mathbf{y}, 0, \ldots, 0])\right\}$. Note that you need to determine the correct numbers of zeros padded to $\mathbf{x}$ and $\mathbf{y}$, respectively. You may use Matlab command `fft` to help you calculate DFT.

Problem 2 (Image Upsampling/Interpolation via Zero Padding, 10') In Lecture 22, you've seen that padding zeros to a signal before calculating its DFT can help improve the density of its discrete Fourier spectrum. In this problem, you will see from a Matlab experiment that, because of the duality/symmetry between the analysis and synthesis equations of DFT, padding zeros in the frequency domain will in turn improve the density in the time domain, effectively upsampling the signal.

You will work on a gray-level images of size 256-by-256. You can read and display the image using the following Matlab commands:

```
img = imread('cameraman.tif');
imshow(img);
```

Consider each row as a signal. You may visualize the frequency domain behaviors from the first row to the last row using the following Matlab commands:

```
t = -128 : 127;
for idx_row = 1 : 256
  row = img(idx_row, :);
  row_fft = fft(row);
  plot(t, abs(fftshift(row_fft)));
```

```
  title(['Row #' int2str(idx_row)]);
  drawnow
  pause(0.01)
end
```

Please write a function named `Upsample1D` that can take as input a row/column signal of the image and a scaling factor $s \geq 1$, and return a upsampled row/column signal. Finally, call the function 512 times to produce upsampled image with $s = 1.5$ and 3. Attach your code and the upsampled images.

Problem 3 (Image Downsampling While Avoiding Aliasing, 10') In Lecture 23, you've seen that in order to avoid aliasing, the highest frequency of the signal content must not be greater than the Nyquist frequency, i.e., $f_s/2$ or $\omega_s/2$, where f_s and ω_s are the sampling frequency and the angular sampling frequency, respectively.

It can be shown that, if we want to downsample a signal by a factor of s, i.e., retaining only one sample point for every s sample points, the DTFT/DFT spectrum will be stretched horizontally by s times and neighboring mirrored spectra will start to overlap, leading to the aliasing phenomenon.

Now, rerun the frequency domain visualization code in Problem 2 and qualitatively determine a maximum scaling factor s_d such that no severe aliasing will happen. Provide your definition of "no severe aliasing will happen." Write your own downsampling/decimation function given an integer-valued `downsample_factor`. Compare the downsampled images by scaling factors s_d, round$(s_d/2)$, and $2s_d$. Comment on their visual quality especially on the visual artifacts.

Problem 4 (Laplace Transform, 10') Determine the Laplace transform and the associated region of convergence for each of the following functions of time. Also provide the pole-zero plot including the region of convergence.

a) $x(t) = e^{-3t}u(t) + e^{-4t}u(t)$

b) $x(t) = e^{-2t}u(t) + e^{-3t}\cos(2t)u(t)$

c) $x(t) = \delta(t) + u(2t)$

d) $x(t) = e^{-3|t|+1}$

Problem 5 (ClassEval, 5') Have you completed ClassEval? It can be found at:

`http://go.ncsu.edu/cesurvey`

Grading: (a) Yes = 5 points, thank you! Please attach the screenshot of the confirmation page. (b) I promise to do it soon = 1 point for good intentions. (c) No = 0 points, a possibly honest answer, but why not spend 5 minutes and get 5 points?

Group Study (1', bonus) Take a screenshot of the whole team with everyone's camera capturing his/her face. One of you will share a window showing the specific homework assignment sheet that you are working on. Include the screenshot in your own homework submission as Problem 6. Your screenshot gets you 1 bonus point; your group members need to do it separately to earn theirs.