ECE 301 (Section 001) Homework 1, Spring 2021

Problem 1 (Complex Numbers)

a) Evaluate and give the answer in both rectangular and polar form. In all cases, assume that $z_1 = 1 + j4$ and $z_2 = -2 + j$. As usual, z^* is the complex conjugate of z.

(a)	z_1^*	(b)	z_2^2	(c)	$z_1 + z_2^*$
(d)	jz_2/z_1^2	(e)	z_1^{-1}	(f)	$z_1/(z_2 + z_1)$
(g)	e^{z_2}	(h)	$z_1 z_1^* z_2 z_2^*$	(i)	$z_1 z_2$

b) Simplify the following numbers into the rectangular form:

(a) $e^{j7\pi}$

(b) $e^{j\pi/3}$

(c) $e^{j13\pi/3}$

(d) $e^{j2021\pi} - e^{j2020\pi}$

Problem 2 (Complex Variable and Function)

a) Let $z = re^{j\theta}$, $r \geq 0$, $\theta \in \mathbb{R}$, be any complex variable. Show that:

(a) $zz^* = r^2$

(b) $z - z^* = 2j\, r \sin\theta$

(c) $(e^z)^* = e^{z^*}$

(d) $z/z^* = e^{j2\theta}$

b) The following complex function $H(\omega)$ is given:

$$H(\omega) = \frac{3}{2 + j\omega}, \quad -\infty < \omega < \infty.$$

Determine and sketch the magnitude and phase of $H(\omega)$.

Problem 3 (Geometric Series) Prove the validity of the following expressions:

a) For $\alpha \in \mathbb{R}$:

$$S_N = \sum_{n=0}^{N-1} \alpha^n = \begin{cases} N, & \alpha = 1, \\ \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1. \end{cases}$$

(Hint: Can you use $S_N - \alpha S_N$ to solve for S_N?)

b) For $|\alpha| < 1$:

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1 - \alpha}.$$

c) For $|\alpha| < 1$:

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2}.$$

(Hint: What happens if you differentiate S_N with respect to α?)

d) For $a, b \in \mathbb{N},\ \alpha \in \mathbb{R}$, and $a \leq b$, find the general expression of:

$$S_{a:b} = \sum_{n=a}^{b} \alpha^n.$$

(Hint: Use the expression of S_N in part a) and express $S_{a:b}$ as the difference between two geometric sums.)

Problem 4 (Logarithms and Decibels)

a) Please simplify the following expressions as much as possible to arrive at primitive log expressions such as $\log_{10}(2)$, $\log_2(\pi)$, $\log_2(3)$, and then use your calculator to evaluate the final numerical results, if applicable.

(a) $\log_{10}(320,000)$

(b) $\log_2(4\pi^2/30)$

(c) $\log(a^{x^2})$

b) A signal $y(t)$ has power that is $195,000,000,000$ times bigger than a signal $x(t)$. What is this power ratio P_y/P_x in decibels?

c) Your cell phone transmits at a power of 23 dBm (this is a common unit of power, and means 23 dB more than a milliWatt). The wireless channel attenuates the signal power by 124 dB, in other words the channel has a (power) gain of -124 dB. Answer the following, realizing that dB is unitless (a ratio of powers) but dBm or dBW (decibels relative to a Watt) do have units (of power).

(a) What is the transmit power in milliwatts?

(b) What is the received power at the base station in dBm?

(c) What is the received power in Watts?

(d) What is the channel gain in linear units? In other words, the channel output $y(t) = gx(t)$ where $x(t)$ is the transmitted signal and $y(t)$ is the received signal. What is the number g in linear units? Hint: we gave you the value of $|g|^2$ in decibels.