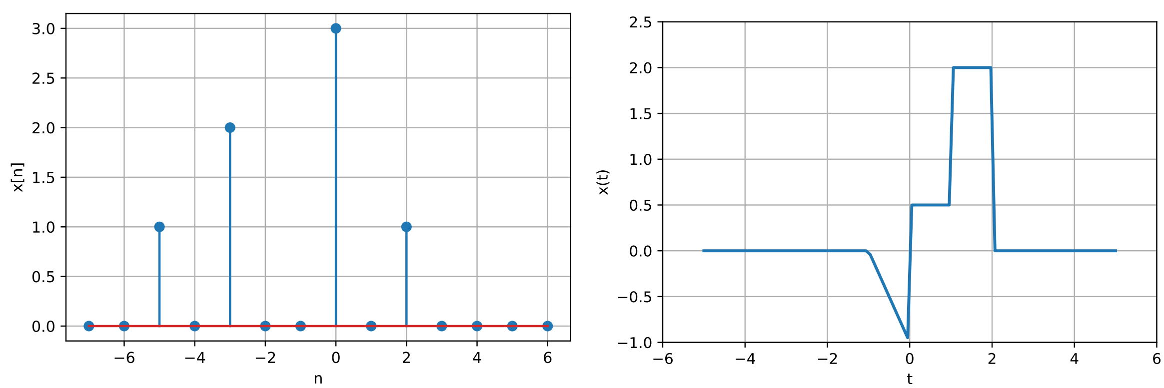
ECE 301 (Section 001) Homework 2, Spring 2021

Problem 1 (Basic Operations on Signals)

a) A discrete-time signal $x[n]$ is shown in Figure 1(a). Sketch carefully each of the following signals:
a) $x[2n]$
b) $x\left[\frac{n}{2}\right]$
c) $x[n]\left(u[n] + \delta[n-2]\right)$

b) A continuous signal $x(t)$ is shown in Figure 1(b). Sketch carefully each of the following signals:
a) $x(t-3)$.
b) $x(1-t/2)$
c) $x(-2t)$
d) $x(t)\left[u(-t) + \delta(t-4)\right]$ (complete this specific question after Monday's lecture)



(a) (b)

Figure 1: (a) Discrete-time signal, $x[n]$. (b) Continuous-time signal, $x(t)$.

Problem 2 (The Periodicity of a Discrete-Time Signal)

a) Determine whether or not the following signals are periodic. If periodic, determine its fundamental period, otherwise explain why it is aperiodic.

(i) $x[n] = \cos(n/6 + \pi/4)$

(ii) $x[n] = e^{j\frac{3\pi}{2}n} + e^{j\frac{5\pi}{3}n}$

b) Use Matlab to plot the real parts of the above signals to verify your analytic results. Append your code and plots in your submission. Add whitespace when necessary to enhance the readability of your code. See Google's Style Guide:
`https://google.github.io/styleguide/cppguide.html#Horizontal_Whitespace`

c) (5', bonus) Consider the periodic discrete-time exponential time signal

$$x[n] = e^{jm(2\pi/N)\,n}.$$

Show that the fundamental period of this signal is

$$N_0 = N/\gcd(m, N),$$

where $\gcd(m, N)$ is the greatest common divisor of m and N—that is, the largest integer that divides both m and N an integral number of times. For example,

$$\gcd(2,3) = 1, \quad \gcd(2,4) = 2, \quad \gcd(8,12) = 4.$$

Note that $N_0 = N$ if m and N have no factors in common.

Problem 3 (Complex Exponentials)

a) What is the signal $y(t) = Ce^{at}u(t)$ as shown in Figure 2? That is, use your new understanding of complex exponential signals to determine A, θ, r, and ω such that $C = Ae^{j\theta}$ and $a = r + j\omega$. Note from the plots that $\text{Re}\{y(0)\} = 0$ and $\text{Im}\{y(0)\} = 2$.

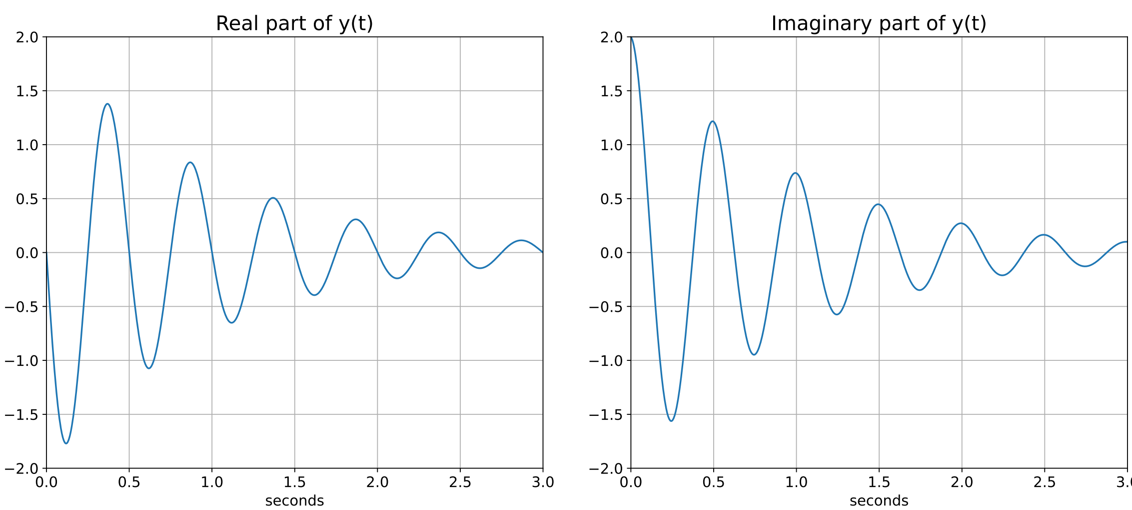


Figure 2: Continuous-time complex exponential, $y(t)$.

b) Determine the discrete time signal $y[n] = C\alpha^n u[n]$ shown in Figure 3. That is, determine A, θ, R, and ω_0 such that $C = Ae^{j\theta}$ and $\alpha = Re^{j\omega_0}$. You can assume $\text{Re}\{y[0]\} = \text{Im}\{y[0]\} = 1/\sqrt{2}$. (Hint: The trigonometric identity, $\cos^2(\phi) + \sin^2(\phi) = 1$, may be helpful.)

Problem 4 (The Fundamental Period of the Sum of Two Signals)

a) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic, and what is the fundamental period of $x(t)$ if it is periodic?

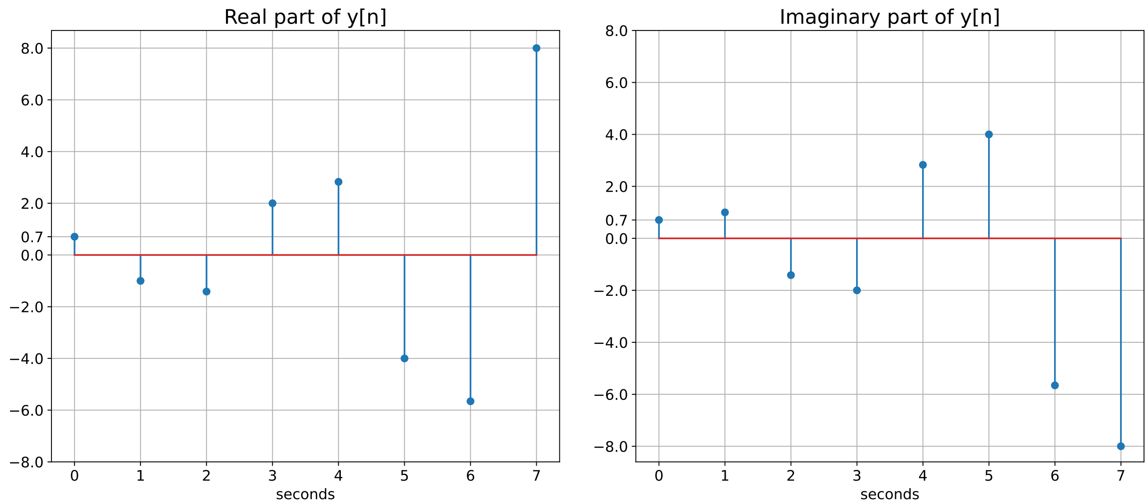


Figure 3: Discrete-time complex exponential, $y[n]$.

b) Let $x_1[n]$ and $x_2[n]$ be periodic sequences with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] = x_1[n] + x_2[n]$ periodic, and what is the fundamental period of $x[n]$ if it is periodic?

Problem 5 (Sifting Property of the Impulse Functions)

a) Simplify the following expressions (complete (c)–(e) after Monday's lecture):

(a) $\left(\frac{\sin(n+\pi)+8}{n^3-2}\right)\delta[n]$

(b) $\left(\frac{\cos(n\pi+\pi)}{-(n+1)^2}\right)\delta[n-3]$

(c) $\left(\frac{\cos t}{t^{64}-4}\right)\delta(t)$

(d) $\left(\frac{5+j3.673\omega+6.345\omega^2}{\omega^2+10}\right)\delta(\omega)$

(e) $\left(\frac{\sin(2k\omega)}{\omega}\right)\delta(\omega)$

b) Evaluate the following integrals and sums (complete (c)–(e) after Monday's lecture):

(a) $\sum_{n=-\infty}^{\infty} \delta[n-6]\gamma^{n-3}\cos\left(\frac{\pi}{12}(6+n)\right)$

(b) $\sum_{n=-\infty}^{\infty} (n^3+n^2+1)\delta[2-n]$

(c) $\int_{-\infty}^{\infty} \delta(t-4)\sin(\pi t)dt$

(d) $\int_{-\infty}^{\infty} \delta(3-t)f(1+t^2)dt$

(e) $\int_{-\infty}^{\infty} \delta(x-2)e^{(x-1)}\cos\left(\frac{\pi}{2}(x^2-5x+4)\right)dx$