ECE 301 (Section 001) Homework 4, Spring 2021

Problem 1 (Linearity Property)

a) Refer to Problem 2 of HW 3 and examine the linearity property for each of the given systems.

b) Refer to Problem 3 of HW 3 and examine the linearity property for each of the given systems.

Problem 2 (Discrete Convolution Basics) Given signal $w[0] = -1$, $w[1] = 2$, $w[2] = -1$, and $w[n] = 0$, elsewhere, and signal $v[0] = 1$, $v[1] = 2$, $v[2] = 3$, $v[3] = 4$, and $v[n] = 0$, elsewhere.

a) Express $w[n]$ and $v[n]$ in the self-referenced form. Plot them in two separate figures. Remember to labels the axes.

b) Decompose input signal $x[n] = w[n]$ into three subsignals, $x_i[n]$ for $i = 1, 2, 3$, send them into an LTI system with impulse response $h[n] = v[n]$, and calculate outputs $y_i[n]$ for $i = 1, 2, 3$ and subsequently $y[n]$ using the graphical approach (see an example from Slides 29–31 of Chapter 2). Clearly define your choices of $x_i[n]$s. Explain how $y_i[n]$s are obtained.

c) Recalculate the output $y[n]$ by using the definition of convolution.

d) Repeat part b) by letting $x[n] = v[n]$ and $h[n] = w[n]$. Are we getting the same result as in b)?

e) Use the definition of convolution, prove that for two generic signals $x[n]$ and $h[n]$, $x[n] * h[n] = h[n] * x[n]$. (You need to clearly demonstrate how a change of variables is done, and how the summation interval of the new dummy variable can be obtained from that of the old dummy variable.) How can this mathematical result be used to explain the finding in d)?

Problem 3 (Discrete Convolution Definition)

a) Compute the convolution $y[n] = x[n] * h[n]$ for $x[n] = \alpha^n u[n]$ and $h[n] = \beta^n u[n]$. Consider the cases $\alpha \neq \beta$ and $\alpha = \beta$.

b) Let $h[n] = \sum_{k=-\infty}^{\infty} \delta[n - 2k]$ be the impulse response of a discrete-time LTI system with input $x[n]$ given by:

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 2. \\ 0, & \text{elsewhere.} \end{cases} \tag{1}$$

(i) Find the output $y[n]$ of the system.

(ii) Is $y[n]$ periodic? If yes, find its fundamental period.

c) (Bonus, 4') Compute the convolution $y[n] = x[n] * h[n]$ for $x[n] = \left(-\frac{1}{2}\right)^n u[n-1]$ and $h[n] = 3^n u[1-n]$.

Problem 4 (The Human Body as an "LTI System") The concentration of an injected (or orally ingested) drug in the blood is commonly modeled as a decaying exponential. This means that if a human body is injected with a drug to time 0, its concentration after n hours is given by $h[n] = \alpha^n u[n]$, where α is defined by the half-life $n_0 > 0$, meaning that $h[n_0] = 0.5h[0]$. You can think of a drug injection as an impulse $x_0[n] = \delta[n]$ and $h[n]$ as the impulse response of the human body, which metabolizes the drug over time.

a) Given that the half-life of a certain drug is 8 hours, what is $h[n]$? Also, find $y[n] = h[n] * x_0[n]$.

b) If instead of an injection, the drug is administered through a steady intravenous (IV) drip starting at time 0, what will be the concentration of the drug as a function of time? Specifically, if $x_1[n] = 0.2u[n]$ find $y[n] = x_1[n] * h[n]$.

c) The drip procedure is not producing a sufficient response. So, a strong extra dose of the drug is injected after 8 hours into the dripping process, namely $x_2[n] = 2\delta[n-8]$. Find the new concentration $y[n]$ as a function of time.

d) Compare the concentrations of the drug in the blood after 16 hours under the three different scenarios, (i) injection at time 0, (ii) drip starting at time 0, (iii) drip starting at time 0 and then injection as modeled by $x_2[n]$.

This homework has four problems.