ECE 301 (Section 001) Homework 8, Spring 2021

Problem 1 (Orthogonal Projection) Consider the set of inconsistent linear equations $\mathbf{Ax} = \mathbf{b}$ given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

- **a)** Find the least-squares solution to these equations.
- **b)** Find the "hat" matrix $\mathbf{H}$. Using Matlab, numerically verify $\mathbf{H} = \mathbf{H}^2$. Argue why.
- **c)** Find the best approximation $\hat{\mathbf{b}} = \mathbf{Hb}$ to $\mathbf{b}$. Find the vector $\mathbf{b}' = (\mathbf{I} - \mathbf{H})\mathbf{b}$ and show numerically that it is orthogonal to $\hat{\mathbf{b}}$.
- **d)** What does the matrix $\mathbf{I} - \mathbf{H}$ represent? If $\mathbf{H}$ is called the "orthogonal projector," can you think of a name for $\mathbf{I} - \mathbf{H}$? Numerically verify $\mathbf{I} - \mathbf{H} = (\mathbf{I} - \mathbf{H})^2$. Argue why.
- **e)** In a 3-dimensional coordinate system, draw the column vectors of matrix $\mathbf{A}$, the column vector space of $\mathbf{A}$, $\mathbf{b}$, $\hat{\mathbf{b}}$, and $\mathbf{b}'$. Make sure that the drawing is reasonably accurate which can reflect the relationship among these quantities.

Problem 2 (Deep Learning with Matlab) In recent updates, Matlab has put together well-guided tutorials for deep learning. This is one set of tutorials on "Deep Learning with Images": `https://www.mathworks.com/help/deeplearning/deep-learning-with-images.html`

Complete the following tutorials by running the example code. Write a concise report consisting of key source code, images, and your explanations.

- **a)** Tutorial "Classify Webcam Images Using Deep Learning."
- **b)** Tutorial "Create Simple Deep Learning Network for Classification."
- **c)** (Bonus, 4') Tutorial "Transfer Learning with Deep Network Designer."

For more tutorials, see the left menu on this page: `https://www.mathworks.com/help/deeplearning/getting-started-with-deep-learning-toolbox.html`

These tutorials may give you ideas about your term project.

Problem 3 (DC Power Supply) One technique for building a DC power supply is to take an AC signal and full-wave rectify it. That is, we put the AC signal $x(t)$ through a system that produces $y(t) = |x(t)|$ as its output.

- **a)** Sketch the input and output waveforms if $x(t) = \cos(t)$. What are the fundamental periods of the input and the output?
- **b)** If $x(t) = \cos(t)$, determine the coefficients of the Fourier series for the output $y(t)$.
- **c)** What is the amplitude of the DC component of the input signal?
- **d)** What is the amplitude of the DC component of the output signal?

Problem 4 (Fourier Series: Analysis/Forward Transform) Find the Fourier series coefficients for each of the following, given that $x(t)$ is a periodic function with period 2π.

a)

$$x(t) = t^3, \quad t \in [-\pi, \pi].$$

Hint:

$$\int t^3 e^{-j\omega kt} dt = \frac{e^{-jkt\omega}\left(jk^3t^3\omega^3 + 3k^2t^2\omega^2 - 6jkt\omega - 6\right)}{k^4\omega^4} + C. \tag{1}$$

b)

$$x(t) = |t|, \quad t \in [-\pi, \pi].$$

Hint: i) The absolute sign goes away when the domain is split into the positive and the negative halves. ii) You will need to use integration by parts.

c) (0', optional) Prove equation (1).

Group Study (1', bonus) Take a screenshot of the whole team with everyone's camera capturing his/her face. One of you will share a window showing the specific homework assignment sheet that you are working on. Include the screenshot in your own homework submission as Problem 5. Your screenshot gets you 1 bonus point; your group members need to do it separately to earn theirs.