

ECE 301 (Section 001) Homework 9, Spring 2021

Problem 1 (Fourier Transform) Compute Fourier transforms for the following signals. You must do the complete computation without relying on the tables.

- a) $x(t) = e^{-2(t-1)}u(t-1)$
- b) $x(t) = 1 - |t|$ for $t \in [0, 1]$ otherwise $x(t) = 0$
- c) $x(t) = e^{-2|t-1|}$
- d) $x(t) = \sum_{n=-\infty}^{\infty} \text{rect}(2t - n)$ [Hint: For this problem, you need to first calculate the Fourier series coefficients, and then proceed by using the FS-to-FT relation on slides 33 of lecture 17.]

Problem 2 (Inverse Fourier Transform and Properties) Compute the inverse Fourier transform for the following signals. You must calculate the results using both i) the direct evaluation method based on the definition of the inverse FT and ii) the table of Fourier transform properties.

- a) $\delta(\omega + 1) + \delta(\omega - 1) + j\delta(\omega + 3) - j\delta(\omega - 3)$
- b) $\text{rect}(\omega/2 + 1)$

Problem 3 (Various Fourier Transform Properties)

- a) Use a procedure similar to that on slide 24 of Lecture 18, prove that differentiation in frequency domain corresponds to multiplication by jt in the time domain, namely,

$$-jtx(t) \xleftrightarrow{\mathcal{FT}} \frac{dX(j\omega)}{d\omega}.$$

- b) Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Fig. 1. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform. [Hint: The subplot (d) may need the result proved in part a).]

Problem 4 (Mathematical Maturity Training, 4' bonus) In Lecture 17, we briefly discussed various sufficient conditions for Fourier transform to exist. I didn't linger on that because, for most engineering applications, we don't counter pathologically behaved functions. Blindly invoking Fourier series/transform formulas is like to be fine. However, for those of you who are mathematically rigorous and/or who want to pursue a graduate degree that usually requires mathematical maturity, understanding pathological examples is important. Please

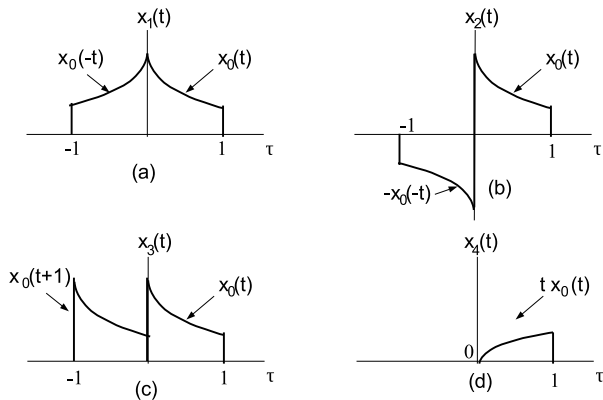


Figure 1: Signals for Fourier transform calculation.

read Sections 3.4 and 4.1.2 of Oppenheim et al.'s book, and then use 3–5 sentences to summarize different sufficient conditions for convergence and use your own words to describe the pathological functions.

Group Study (1', bonus) Take a screenshot of the whole team with everyone's camera capturing his/her face. One of you will share a window showing the specific homework assignment sheet that you are working on. Include the screenshot in your own homework submission as Problem 5. Your screenshot gets you 1 bonus point; your group members need to do it separately to earn theirs.