Problem 1 (Windowed Time-Domain Signals) Find and plot the Fourier transform of the following windowed sinusoidal signals.

    a)

$$x(t) = \begin{cases} \cos(10t), & -10 \leq t \leq 10. \\ 0, & \text{elsewhere.} \end{cases}$$

    b)

$$x(t) = \begin{cases} \cos(10t), & 0 \leq t \leq 10. \\ 0, & \text{elsewhere.} \end{cases}$$

    Hint: Use the multiplicative property of the Fourier transform.

Problem 2 (Lowpass Filter) A lowpass filter $H(\omega)$ has the frequency response shown in Figure 1.
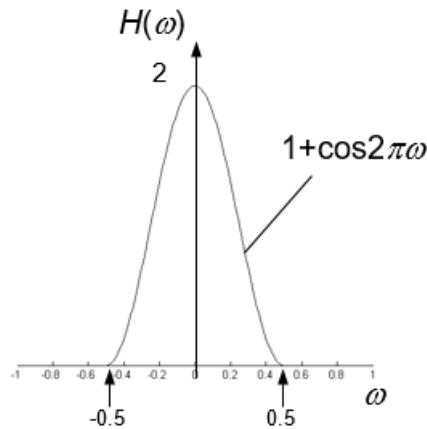

Figure 1: Frequency response of the lowpass filter $H(\omega)$.

    a) Compute the impulse response $h(t)$ of the filter.

    b) Compute the response $y(t)$ when the input is $x(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$, $-\infty < t < \infty$.

    c) Compute the response $y(t)$ when $x(t) = \text{sinc}\left(\frac{t}{4\pi}\right)$, $-\infty < t < \infty$.

    Note: Refer to version 2 of Lecture 18 slides for the Fourier table.

Problem 3 (DTFT and Inverse DTFT) Solve the following using the definitions *or* the properties.

    a) $\mathcal{F}\{u[n-2] - u[n-6]\}$

b) $\mathcal{F}\{2^n \sin(\pi n/4)u[-n]\}$

c) $\mathcal{F}\{\sin(\pi n/2) + \cos(7\pi n/3)\}$

d) $\mathcal{F}^{-1}\left\{\frac{e^{-j\omega} - \frac{1}{3}}{1 - \frac{1}{3}e^{-j\omega}}\right\}$

Problem 4 (5', bonus) (Computational Complexity in Time Domain vs. in Frequency Domain) In class, you've seen that calculating the convolution between two time-domain sinc functions in the frequency domain can significantly lower the computation cost. Argue under what scenario time-domain calculation can significantly lower the computation cost. Give a concrete example in terms of two signals.

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.