ECE 301 (Section 001) Homework 1, Spring 2022

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Problem 1 (Complex Numbers)

a) Evaluate and give the answer in both rectangular and polar form. In all cases, assume that $z_1 = 1 + j4$ and $z_2 = -2 + j$. As usual, z^* is the complex conjugate of z.

(a) z_1^*	(b) z_2^2	(c) $z_1 + z_2^*$
(d) jz_2/z_1^2	(e) z_1^{-1}	(f) $z_1/(z_2 + z_1)$
(g) e^{z_2}	(h) $z_1 z_1^* z_2 z_2^*$	(i) $z_1 z_2$

b) Simplify the following numbers into the rectangular form:

(a) $e^{j7\pi}$

(b) $e^{j\pi/3}$

(c) $e^{j13\pi/3}$

(d) $e^{j2021\pi} - e^{j2020\pi}$

Problem 2 (Complex Variable and Function)

a) Let $z = re^{j\theta}$, $r \geq 0$, $\theta \in \mathbb{R}$, be any complex variable. Show that:

(a) $zz^* = r^2$

(b) $z - z^* = 2j\, r \sin\theta$

(c) $(e^z)^* = e^{z^*}$

(d) $z/z^* = e^{j2\theta}$

b) The following complex function $H(\omega)$ is given:

$$H(\omega) = \frac{3}{2 + j\omega}, \quad -\infty < \omega < \infty.$$

Determine and sketch the magnitude and phase of $H(\omega)$.

Problem 3 (Geometric Series) (You may do it after attending Friday's discussion.) Prove the validity of the following expressions:

a) For $\alpha \in \mathbb{R}$:

$$S_N = \sum_{n=0}^{N-1} \alpha^n = \begin{cases} N, & \alpha = 1, \\ \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1. \end{cases}$$

(Hint: Can you use $S_N - \alpha S_N$ to solve for S_N?)

b) For $|\alpha| < 1$:

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}.$$

c) For $|\alpha| < 1$:

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2}.$$

(Hint: What happens if you differentiate S_N with respect to α?)

d) For $a, b \in \mathbb{N}$, $\alpha \in \mathbb{R}$, and $a \leq b$, simplify the following summation:

$$S_{a:b} = \sum_{n=a}^{b} \alpha^n.$$

(Hint: Use the expression of S_N in part a) and express $S_{a:b}$ as the difference between two geometric sums.)

Problem 4 (Logarithms and Integration)

a) Please simplify the following expressions as much as possible to arrive at primitive log expressions such as $\log_{10}(2)$, $\log_2(\pi)$, $\log_2(3)$, and then use your calculator to evaluate the final numerical results, if applicable.

(a) $\log_{10}(320, 000)$
(b) $\log_2(4\pi^2/30)$
(c) $\log(a^{x^2})$

b) A signal $y(t)$ has power that is $195, 000, 000, 000$ times bigger than a signal $x(t)$. What is this power ratio P_y/P_x in decibels?

c) Compute the following integral:

$$y(t) = \int_0^t x(\tau)\, d\tau, \quad t \geq 0. \tag{1}$$

A graph of $x(t)$ is given below. The line segments are straight with $x(0) = 0, x(1) = 1$, and $x(2) = 0$. Note that your solution will be a piecewise function.