ECE 301 (Section 001) Homework 3, Spring 2022

Problem 1 (Basic Operations on Signals)

a) A discrete-time signal $x[n]$ is shown in Figure 1(a). Sketch carefully each of the following signals by hand:

i) $x[2n]$

ii) $x\left[\frac{n}{2}\right]$

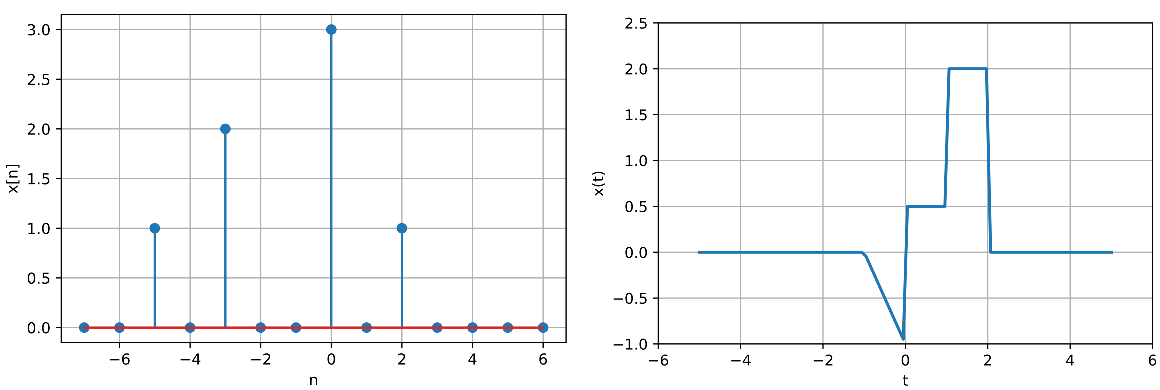
iii) $x[n]\left(u[n]+\delta[n-2]\right)$

b) A continuous signal $x(t)$ is shown in Figure 1(b). Sketch carefully each of the following signals by hand:

i) $x(t-3)$

ii) $x(1-t/2)$

iii) $x(t)\left[u(-t)+\delta(t-4)\right]$



(a) (b)

Figure 1: (a) Discrete-time signal, $x[n]$. (b) Continuous-time signal, $x(t)$.

Problem 2 (Sifting Property of the Impulse Functions)

a) Simplify the following expressions:

i) $\left(\frac{\sin(n+\pi)+8}{n^3-2}\right)\delta[n]$

ii) $\left(\frac{\cos(n\pi+\pi)}{-(n+1)^2}\right)\delta[n-3]$

iii) $\left(\frac{\cos t}{t^{64}-4}\right)\delta(t)$

iv) $\left(\frac{\sin(2k\omega)}{\omega}\right)\delta(\omega)$

b) Evaluate the following integrals and sums:

i) $\sum_{n=-\infty}^{\infty} \delta[n-6]\gamma^{n-3} \cos\left(\frac{\pi}{12}(6+n)\right)$

ii) $\int_{-\infty}^{\infty} \delta(t-4)\sin(\pi t)dt$

iii) $\int_{-\infty}^{\infty} \delta(3-t)f(1+t^2)dt$

iv) $\int_{-\infty}^{\infty} \delta(x-2)e^{(x-1)} \cos\left(\frac{\pi}{2}(x^2-5x+4)\right) dx$

Problem 3 (Impulse Function and Step Function)

a) Find and sketch the first derivatives of the following signals:

i) $x(t) = u(t) - u(t-a)$, $a > 0$,

ii) $x(t) = t[u(t) - u(t-a)]$, $a > 0$.

iii) $x(t) = \text{sign}(t) = \begin{cases} 1, & t \geq 0, \\ -1, & t < 0. \end{cases}$

b) Given that the $\delta(\cdot)$ is the unit impulse function, prove the following property:

$$\int_{-\infty}^{\infty} \delta(ax)dx = \int_{-\infty}^{\infty} \frac{\delta(t)}{|a|}dt = \frac{1}{|a|},$$

where a is a scalar. Can you claim that $\delta(ax) = \frac{1}{|a|}\delta(t)$? If yes, in what sense? If no, why.

Problem 4 (Properties of Discrete Systems) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y[n]$ denotes the system output and $x[n]$ denotes the system input.

a) $y[n] = x[n]\,x[n-1]\,x[n+1]$

b) $y[n] = \begin{cases} x[n-1], & n \leq 0, \\ x[n+1], & n > 0. \end{cases}$

c) $y[n] = \cos(x[n])$

Problem 5 (Properties of Continuous Systems) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y(t)$ denotes the system output and $x(t)$ is the system input.

a) $y(t) = (t-2)\,x(t+2)$

b) $y(t) = x(t-1) + x(4-t)$

c) $y(t) = x\big(\sin(t)\big)$

d) (Bonus, 7') $y(t) = \begin{cases} 0, & x(t) < 1, \\ \int_0^1 x(t-\tau)d\tau, & x(t) \geq 1. \end{cases}$

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.