ECE 301 (Section 001) Homework 5
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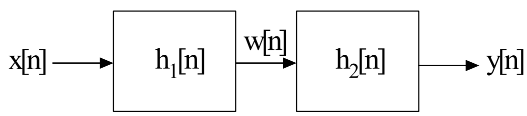
TA in Charge: Fin Amin

Problem 1 (Bonus) (Associative Property of Convolution)

a) Consider two LTI systems with the impulse responses

$$h_1[n] = \left(-\frac{1}{2}\right)^n u[n],$$
$$h_2[n] = u[n] + \frac{1}{3}u[n-1].$$

These two systems are cascaded as shown in the following figure. Let $x[n] = u[n]$.



i) (Bonus, 6') Prove that $u[n] * u[n] = (n+1)u[n]$ and $u[n] * u[n-1] = nu[n-1]$.

ii) (Bonus, 10') Verify the associative property of convolution by showing that $y[n] = (x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$. [**Hint:** Try to apply the distributive property of convolution for the most parts of the proof. At the very last step, apply the definition of convolution or use the graphical approach to obtain the final solution.)

b) (Bonus, 4') Prove the following associative property of convolution from the definition:

$$[x(t) * h(t)] * g(t) = x(t) * [h(t) * g(t)].$$

Problem 2 (Applications of Convolution in Networking and Communications)

a) A lossless computer network with a reflection is found to have impulse response

$$h(t) = \delta(t - 50 \text{ nsec}) - 0.7\delta(t - 100 \text{ nsec}). \tag{1}$$

A rectangular pulse $x(t) = u(t) - u(t-T)$ is input into the network and $y(t) = h(t) * x(t)$ is received. Plot $y(t)$ over $0 \le t \le 0.3$ μsec if the pulse length T is given by $T = 70$ nsec.

b) A transmitter $s(t)$ inputs a signal into $c(t)$, a wireless channel having reflections. The channel has the following impulse response:

$$c(t) = A_d\delta(t - t_d) + A_1\delta(t - t_d - t_1), \tag{2}$$

where $A_d = 1$, $t_d = 16.67$ nsec, $A_1 = 0.95$, and $t_1 = 2$ nsec. The sinusoidal carrier frequency is given by $f_c = 0.5$ GHz. The transmitted signal is

$$s(t) = \begin{cases} \sin(2\pi f_c t), & 0 \le t \le 3T_p, \\ 0, & \text{else}, \end{cases} \tag{3}$$

where T_p is one period of the carrier. (i) Compute and plot the impulse response $c(t)$ of this channel. (ii) The received signal $r(t)$ is given by the convolution $r(t) = c(t) * s(t)$. Plot $r(t)$ over $0 \leq t \leq 50$ nsec.

Problem 3 (Evaluate Convolution) Perform convolutions of the following functions. Use hand drawings to determine the intervals. You will need to show intermediate calculation steps to get full points.

a) (6') $x(t) = \text{rect}(t)$ and $h(t) = \text{rect}(2t - 1/2)$.

b) (6') $x(t) = e^{-2t}u(t)$ and $h(t) = e^{-3t}u(t)$.

c) (8') $x(t) = e^{4}[u(t+3) - u(t)]$, $h(t) = e^{-3t}[u(t-1) - u(t-2)]$.

Problem 4 (Implement Convolution) Use Matlab to implement the convolution between

$$x[n] = 0.5^n \cos(n\pi/4)(u[n] - u[n-10]), \text{ and}$$
$$h[n] = 0.6^n \sin(n\pi/3)(u[n] - u[n-10]).$$

Plot the input, output, and the impulse response on the same graph (2'). Use different colors for different signals (2'). (Type "`help plot`" to see how to do it.) Use `legend()` to create a descriptive label for each plotted signal (2'). Properly label the axes (2'). Append your source code to the submission (12'). Note that you need to implement the convolution operation by yourself, instead of using the built-in function conv() from Matlab. You can use conv() to verify the correctness of your implementation. Inserting zeros to the beginning or the end of the vector x and/or vector h may help you avoid negative indexing issues.

Problem 5 (More on Convolution)

a) Let the impulse response of an LTI system be

$$h(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT),$$

for a given T. Let $y(t)$ be the convolution of $h(t)$ with the input $x(t)$, i.e., $y(t) = x(t) * h(t)$. Suggest two different input signals $x(t)$ that give an output of $y(t) = 1$, $\forall\, t$. [**Hint:** $y(t) = 1$ can be thought of as a horizontal concatenation of infinitely many rectangular windows.]

b) Determine the output of an LTI system when the input and the impulse response are given by

$$x(t) = \text{rect}\left(\frac{t}{3} - \frac{1}{6}\right) \text{ and } h(t) = e^{-(t-5)}u(t-5), \text{ respectively.}$$

Recall that $\text{rect}(t) = 1$ for $t \in (-0.5, 0.5)$ and is zero elsewhere.

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.