ECE 301 (Section 001) Homework 7
Spring 2022, Dr. Chau-Wai Wong
TA in Charge: Fin Amin

Problem 1 (Projection and PCA) You are given a data matrix

$$\mathbf{X} = \begin{bmatrix} -0.92 & 1.09 & -1.35 & 2.06 & -0.60 & -0.28 \\ -0.08 & 1.15 & -1.67 & 1.08 & -1.14 & 0.66 \end{bmatrix}.$$

and its principal component $\mathbf{u} = [0.7474,\ 0.6644]^T$.

a) Plot the data points using circles and the principal component using a line segment with one end at $[0, 0]^T$ on a 2D plane using Matlab.

b) Calculate the projection c_i for each data point $\mathbf{x}_i$ to $\mathbf{u}$. Note that c_i can be negative.

c) Calculate the sample variance of $\{c_i\}_{i=1}^n$, namely,

$$\widehat{\text{Var}}\left(\{c_i\}\right) = \frac{1}{n-1} \sum_{i=1}^{n} \left(c_i - \overline{c}\right)^2,$$

where $\overline{c} \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} c_i$ is the sample mean.

d) Repeat b) and c) using another unit vector $\mathbf{u}_a = [0.8,\ 0.6]^T$. Is the newer sample variance smaller than that in b)? Can you explain why?

Problem 2 (Orthogonal and Unitary Matrices)

You have learned in class that a real-valued *orthogonal matrix* $\mathbf{P}$ has the property that $\mathbf{P}^T\mathbf{P} = \mathbf{P}\mathbf{P}^T = \mathbf{I}$. In signal processing, oftentimes signals are represented in complex values. We can define for a complex-valued square matrix a similar concept called the *unitary matrix*. A unitary matrix $\mathbf{Q}$ satisfies the property that $\mathbf{Q}^H\mathbf{Q} = \mathbf{Q}\mathbf{Q}^H = \mathbf{I}$, where "^{H}" is the Hermitian operator that is a combination of both the transpose, "^{T}," and the complex conjugate, "$*$."

a) The discrete Fourier transform (DFT) matrix for a signal of length 3 is defined as $\mathbf{Q}_3 = [q_{kn}]_{k,n\in\{0,1,2\}}$, where $q_{kn} = \frac{1}{\sqrt{3}}\exp(-j\frac{2\pi}{3}kn)$. Explicitly write out this 3-by-3 matrix. Simplify each entry but do not evaluate numerically the complex exponentials.

b) Verify that $\mathbf{Q}_3$ is a unitary matrix.

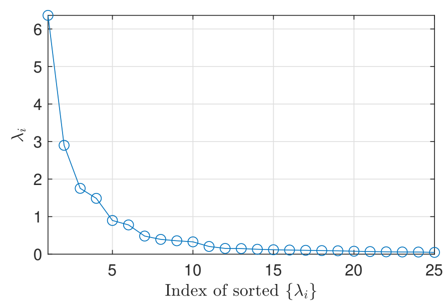
c) Compute the forward discrete Fourier transform for signal $\mathbf{x} = [1.01, 0.99, 0.97]^T$ to obtain a transformed signal $\mathbf{z} = \mathbf{Q}_3\,\mathbf{x}$, where $\mathbf{z} = [z_1, z_2, z_3]^T$. How large are the norms for z_1, z_2, and z_3?

d) Now, zero out z_3 to obtain a new vector $\mathbf{z}_{\text{compressed}}$. Compute the inverse transform using $\hat{\mathbf{x}} = \mathbf{Q}_3^H\,\mathbf{z}_{\text{compressed}}$. Is $\hat{\mathbf{x}}$ similar to $\mathbf{x}$? Can you guess why?

Problem 3 (PCA on Downsampled Yale Face Database) In this problem, we will explore PCA as a visualization tool for Yale Face Database. Download the `.m` files and the database. Extract the face image files into a folder named `yalefaces` and put the `.m` files at the same level of the folder. Call this folder "problem3/". Open Matlab, change your "Current Folder" to "problem3/", and open `main_pca_visualization.m`.

a) Run the code corresponding to this part only, describe the data structure of variable `img_buffer`. Set `preview_img_flag` to `1`, re-run the code to visually inspect the whole database.

b) Complete Matlab function `[V, Lambda_mat] = PcaViaKlt(data)` by implementing PCA using eigendecomposition on a sample covariance matrix of the face data. The detailed information about the input and outputs are given in the comments of the incomplete function. You may use built-in function `eig` for eigendecomposition. If your implementation is correct, after running the code of b), you will obtain a plot similar to the following.



c) Run the code of c) to visualize a few dominating eigenvectors. Comment on whether they reflect some characteristics of the faces you saw in a).

d) The code of d) projects each face image (coming from one of the four selected classes) onto a 2D space. Comment on PCA's data visualization performance in this specific example.

Problem 4 (Deep Learning with Matlab) In recent updates, Matlab has put together well-guided tutorials for deep learning. This is one set of tutorials on "Deep Learning with Images":
`https://www.mathworks.com/help/deeplearning/deep-learning-with-images.html`

Complete the following tutorials by running the example code:

"Classify Webcam Images Using Deep Learning."

Write a concise report consisting of key source code, images, and your explanations.

For more tutorials, see the left menu on this page: `https://www.mathworks.com/help/deeplearning/getting-started-with-deep-learning-toolbox.html`

These tutorials may give you ideas about your term project (if assigned).

Problem 5 (Mid-Semester Class Evaluation, Bonus 10') To get the bonus points, please take a screenshot of the confirmation page after submitting the survey and use the screenshot as the answer to this problem. Survey link:

`https://forms.gle/cX15yeUhCtdHQhiY7`

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.