Problem 1 (Fourier Transform) Compute Fourier transforms for the following signals. You must do the complete computation without relying on the tables.

a) $x(t) = e^{-2(t-1)}u(t-1)$

b) $x(t) = e^{-|t+2|/3}$ (Explicitly show how the absolute sign is removed.)

c) $x(t) = \text{rect}(2t+1)$ (Final result must be represented in form of a sinc function.)

d) $x(t) = 1+t$ for $t \in [-1,0]$ otherwise $x(t) = 0$ (Show the details of integration by parts.)

Problem 2 (Inverse Fourier Transform and Properties) Compute the inverse Fourier transform for the following signals. You must calculate the results using both i) the direct evaluation method based on the definition of the inverse FT and ii) the table of Fourier transform properties.

a) $\delta(\omega+1) + \delta(\omega-1) + j\delta(\omega+3) - j\delta(\omega-3)$

b) $\text{rect}(3\omega - 2)$

Problem 3 (Various Fourier Transform Properties)

a) Use a procedure similar to that on slide 64 of Lecture 19, prove that differentiation in frequency domain corresponds to multiplication by jt in the time domain, namely,

$$-jtx(t) \overset{\mathcal{FT}}{\longleftrightarrow} \frac{dX(j\omega)}{d\omega}.$$

b) Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Fig. 1. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform. [Hint: The subplot (d) may need the result proved in part a).]

Problem 4 (Bonus, 20') (Mathematical Maturity Training. This problem is difficult, so proceed with caution.) In Lecture 19, I provided a few supplemental slides discussing various sufficient conditions for Fourier transform to exist. I didn't spend time on that because, for most engineering applications, we don't counter pathologically behaved functions; blindly invoking Fourier series/transform formulas is likely to be fine. However, for those of you who are mathematically rigorous and/or who want to pursue a graduate degree that usually
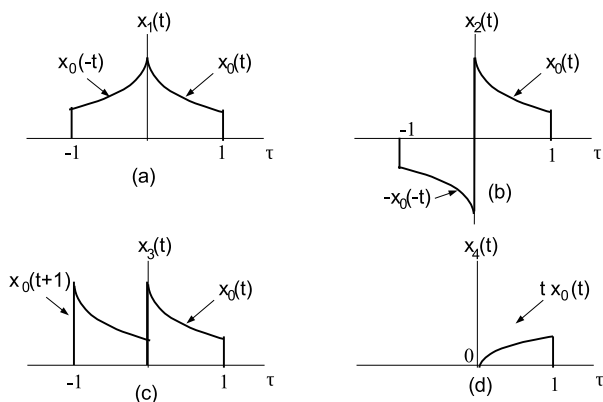
Figure 1: Signals for Fourier transform calculation.

requires mathematical maturity, understanding pathological examples is important. Please read Sections 3.4 and 4.1.2 of Oppenheim et al.'s book, and then use 3–5 sentences to summarize different sufficient conditions for convergence and use your own words to describe the pathological functions.

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.