ECE 301 (Section 001) Homework 10
Spring 2023, Dr. Chau-Wai Wong
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Problem 1 (DC Power Supply) One technique for building a DC power supply is to take an AC signal and full-wave rectify it. That is, we put the AC signal $x(t)$ through a system that produces $y(t) = |x(t)|$ as its output.

- **a)** Sketch the input and output waveforms if $x(t) = \cos(t)$. What are the fundamental periods of the input and the output?
- **b)** If $x(t) = \cos(t)$, determine the coefficients of the Fourier series for the output $y(t)$.
- **c)** What is the amplitude of the DC component of the input signal?
- **d)** What is the amplitude of the DC component of the output signal?

Problem 2 (Fourier Series: Analysis/Forward Transform) Find the Fourier series coefficients for each of the following, given that $x(t)$ is a periodic function with period 2π.

- **a)**

$$x(t) = t^3, \quad t \in [-\pi, \pi].$$

Hint:

$$\int t^3 e^{-j\omega k t} dt = \frac{e^{-jkt\omega}\left(jk^3t^3\omega^3 + 3k^2t^2\omega^2 - 6jkt\omega - 6\right)}{k^4\omega^4} + C. \tag{1}$$

- **b)**

$$x(t) = |t|, \quad t \in [-\pi, \pi].$$

Hint: i) The absolute sign goes away when the domain is split into the positive and the negative halves. ii) You will need to use integration by parts.

- **c)** (2', optional) Prove equation (1).

Problem 3 (Fourier Transform) Compute Fourier transforms for the following signals. You must do the complete computation without relying on the tables.

- **a)** $x(t) = e^{-2(t-1)}u(t-1)$
- **b)** $x(t) = e^{-|t+2|/3}$ (Explicitly show how the absolute sign is removed.)
- **c)** $x(t) = \text{rect}(2t+1)$ (Final result must be represented in form of a sinc function.)
- **d)** $x(t) = 1 + t$ for $t \in [-1, 0]$ otherwise $x(t) = 0$ (Show the details of integration by parts.)

Problem 4 (Inverse Fourier Transform and Properties) Compute the inverse Fourier transform for the following signals. You must calculate the results using both i) the direct evaluation method based on the definition of the inverse FT and ii) the table of Fourier transform properties.

a) $\delta(\omega+1)+\delta(\omega-1)+j\delta(\omega+3)-j\delta(\omega-3)$

b) $\text{rect}(3\omega-2)$

Problem 5 (Eigen-signals/functions of an LTI System, bonus 5') (Attempt the problem after the 3/20 lecture) In Lecture 17, you were told that $e^{j\omega_0 t}$ is an eigen-signal of an LTI system $h(t)$. In other words, when the LTI system operates on an input signal $x(t) = e^{j\omega_0 t}$ [or $x(t)$ is sent into the LTI system], the output $y(t)$ is merely a scaled version of $x(t)$ for all $t \in \mathbb{R}$. Show that the scaling factor is

$$H(j\omega_0) = \int_{-\infty}^{\infty} h(t)e^{-j\omega_0 t}\,dt. \tag{2}$$

Recall that the input–output relation of an LTI system is related by $y(t) = h(t) * x(t)$.

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.