ECE 301 (Section 001) Homework 7
Spring 2023, Dr. Chau-Wai Wong
TA in Charge: Devadharshini Ayyappan

Problem 1 (Properties of LTI Systems) Determine whether the following LTI systems are causal, memoryless, and stable, respectively. Justify your answers.

    a) $h(t) = e^{-2t}u(t) + e^{t/100}u(t-1)$

    b) $h[n] = (n+1)2^n u[n]$

    c) $h[n] = 4^n u[2-n]$

    d) $h(t) = e^{-2t}\left[u(t) - u(t-1)\right].$

Problem 2 (Vector and Matrix Refresh) Seven data points are arranged as columns of a data matrix $\mathbf{X}$ given as follows:

$$\mathbf{X} = \begin{bmatrix} 2 & 1 & 0 & 0 & -1 & 0 & -2 \\ 2 & 0 & 1 & 0 & 0 & -1 & -2 \end{bmatrix}.$$

    a) Draw all data points on a 2D plane by hand. Properly label the two axes. Clearly mark important axis tick values to facilitate a precise graphical description of the data.

    b) Consider each point as a vector. Calculate the angle (in °) between $[2\ 2]^T$ and the five other points (excluding $[0\ 0]^T$), respectively, using the inner/dot product formula that involves the angle. Note that the angle between two vectors can be negative.

    c) Calculate the matrix outer product for $\mathbf{X}$, namely, $\mathbf{R} = \mathbf{X}\mathbf{X}^T$. Show the intermediate steps of calculating each element of the 2-by-2 matrix $\mathbf{R}$.

    d) The matrix outer product can also be calculated via $\mathbf{R} = \sum_{i=1}^{7} \mathbf{x}_i\mathbf{x}_i^T$, where $\mathbf{x}_i$ is the ith column of $\mathbf{X}$. Evaluate the numerical result. Note that each $\mathbf{x}_i\mathbf{x}_i^T$ is a 2-by-2 matrix.

    e) Now, consider each column of $\mathbf{X}$ as a *single* block-entry of the matrix. Rewrite $\mathbf{X}$ and $\mathbf{X}^T$ into the form of the vector of blocks-entries, respectively. Use the vector multiplication rule to show that $\mathbf{R} = \sum_{i=1}^{7} \mathbf{x}_i\mathbf{x}_i^T$.

Problem 3 (Linearly independence, Basis, and Vector Space)

    a) Are vectors $[1\ 2], [4\ 5]$, and $[7\ 8]$ linearly independent? What about $[1\ 2\ 0], [1\ -1\ 1]$, and $[0\ 0\ 1]$? Justify your answers.

    b) You are given a vector space $V = \text{span}\{[-1\ 0\ 0], [0\ -1\ 0]\}$.

        (i) Express V in a set representation.

        (ii) Can you find a basis for V?

        (iii) Are $[5\ 8\ 0], [8\ 0\ 5]$, and $[0\ 5\ 8]$ in vector space V? Is yes, what are the coefficient for each vector of the basis you found in (ii)?

        (iv) Draw all points of (iii) in a 3D coordinate. Illustrate vector space V using a plane formed by the vectors of the basis.

c) (Bonus, 5') Let

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & -1 & 0 \\ -1 & -1 & 2 & -3 \\ 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

What is the dimension of the column vector space of $\mathbf{A}$? What is the rank of $\mathbf{A}$?

Problem 4 (Machine Learning Intro Video) Watch this 10-minute video:

$$\text{https://youtu.be/YH1gdd8kJXo}$$

Write 6–8 sentences to concisely summarize machine learning and/or artificial intelligence.

Problem 5 (Mid-Semester Class Evaluation, Bonus 10') To get the bonus points, please take a screenshot of the confirmation page after submitting the survey and use the screenshot as the answer to this problem. Survey link:

$$\text{https://forms.gle/Wp4qZRzQniD2uR317}$$

Group Study (1', bonus) Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet that you are working on. In-Person: Take a selfie with all group members' faces in the photo. Capture the homework assignment sheet in the photo.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.