ECE 301 (001) Linear Systems
Spring 2023 Midterm Exam 1
Instructor: Dr. Chau-Wai Wong

This is a closed-book, closed-lecture notes exam. One-sided, letter-sized, handwritten cheat sheet is allowed. Calculators are not allowed. You are required to answer ***four complete problems*** of your choice. (If you provide answers to all five problems, points will be assigned only for four randomly chosen problems.) Submission instructions for in-class students:

1. Once the instructor says "stop writing," please use your mobile phone to "scan" every answer sheet. (After scanning, you must turn in the physical copy of your answer sheets to the instructor, or you will receive zero for this exam.)

2. Do not forget to scan those pages on the back if you wrote answers on them. Double check the scanned pages one by one on your phone to make sure they are properly captured.

3. You have a time window of 6 hours after the exam to upload your answers to Gradescope. Upload your scanned pages as a PDF file to Gradescope and tag each page.

4. There will be a 10-point penalty for every late hour after the submission deadline.

5. Turn in the physical copy of your answer sheets to the instructor in the front of the classroom.

If you take the exam at a university testing center, proctoring staff will scan your papers and send them to Dr. Wong. Dr. Wong will later forward the scanned PDF to you, and you will be responsible for uploading the PDF file to Gradescope and tagging each page within 24 hours of receiving Dr. Wong's email.

Problem 1 (25 pts) [Geometric Series] Let

$$S = \alpha^{m-3} - \alpha^{m-2} + \cdots + \alpha^{m+9} - \alpha^{m+10},$$

where $m \in \mathbb{Z}$. Derive the analytic form of S without the help of the geometric series formula. Show intermediate steps clearly. Remember to deal with different cases separately.

Problem 2 (25 pts) [Operations on Signals]

(a) You are given a discrete-time signal $x[n] = \delta[n+1] - 2\delta[n] + 5\delta[n+3] - \delta[n-6]$, $n \in \mathbb{Z}$. Sketch carefully the following signals.

$$y[n] = x[2n-1].$$

Partial points will be given for intermediate calculations/manipulations you write down.

(b) You are given signal $x(t) = 2\cos(t)\,[u(t+\pi/2) - u(t-\pi/2)]$, $t \in \mathbb{R}$.

(i) Sketch $x(t)$. Clearly provide important tick values on the t-axis and x-axis to facilitate a precise graphical description of $x(t)$.

(ii) Calculate the first derivative of $x(t)$. Note that $(uv)' = u'v + uv'$, $(\sin x)' = \cos x$, and $(\cos x)' = -\sin x$.

(iii) Sketch the first derivative of $x(t)$ using (1) the result of (ii), or (2) your observation from the graph you sketched for (i). If you proceed via route (2), justify in words how you obtained the various components in your plot.

Problem 3 (25 pts) [Periodicity and Sifting Property]

(a) Determine whether or not the following signals are periodic. If periodic, determine its fundamental period, otherwise explain why it is aperiodic. (If you use off-the-shelf formulas, make sure they are applied correctly. You may also try to find the period by starting from the definition, which allows us to give you partial credits even if the final result is wrong.)

(i) $x(t) = \cos(2t/7 + \pi/3)$.

(ii) $x[n] = \sin\left(\frac{2024}{2023}\pi n\right) + \exp\left(j\frac{7}{2}\pi n\right)$.

(b) Simplify/evaluate the following expressions. To save your time, there is no need to copy the following equations to your answer sheet.

(i) $\sum_{n=-\infty}^{\infty} (n^3 + n^2 + 1)\delta[n+1]$.

(ii) $\int_{-\infty}^{\infty} \delta(x-1)e^{x-1} \cos\left(-\frac{\pi}{32}(x^2 - 5x + 4)\right) dx$.

Problem 4 (25 pts) [Convolution and Correlation]

You are given $x(t) = e^{-2t}u(t-2)$ and $h(t) = u(t+5)$ for $t \in \mathbb{R}$. Using a graphical approach to determine how the interval $t \in \mathbb{R}$ should be divided into subintervals on which $y(t)$ will have different analytic expressions. Explicitly **specify the range of each subinterval**. For each subinterval, set up an integral with **proper lower and upper integration limits**, but do NOT proceed to evaluate the numerical result of the integral. For sanity check, the union of all subintervals must be $\mathbb{R}$.

(a) When $y(t)$ is the *covolution* between $x(t)$ and $h(t)$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau.$$

(b) When $y(t)$ is the *correlation* between $x(t)$ and $h(t)$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(\tau - t)\, d\tau.$$

Problem 5 (25 pts) [Time-Invariance and Linearity]

(a) Determine whether the following systems with input x and output y are time-invariant. You must justify with mathematical expressions and/or in words at each intermediate step.

(i) $y(t) = 4t\, x(t-3)$.

(ii) $y[n] = 5x[2n] - 1$.

(b) Determine whether the systems in (a) are linear. You must justify with mathematical expressions and/or in words at each intermediate step.