ECE 301 (Section 001) Homework 10
Spring 2024, Dr. Chau-Wai Wong
TA in Charge: Prasun Datta

Problem 1 (10 pts) Please attach the full scan/photo of your graded Quiz 4.

If your score is less than 100% (including those who missed the quiz), you may recover at most 50% the points you lost by completing the following question.

If you were sick, attach the supporting document and complete the following question to obtain at most 10 points.

Given input signal $x(t) = \text{rect}((t-2)/4)/4$ and impulse response $h(t) = \text{rect}((t-1.5)/3)/3$, determine output $y(t) = x(t) * h(t)$ by following the steps below:

a) Draw $x(\tau)$ and $h(t-\tau)$ with respect to τ on two 2-d planes, respectively.

b) Use the graphical approach to determine all subintervals on which $y(t)$ will have different expressions.

c) Evaluate $y(t)$ for both **ends** of each subinterval using the geometric/graphical approach. (You will not receive points for using the analytic/calculus approach. This requirement is different from that of the original quiz.)

d) Draw all $(t, y(t))$ pairs obtained in the previous step on a 2-d plane. Connect neighboring points to obtain the graphical representation of the $y(t)$.

Problem 2 (Eigen-signals/functions of an LTI System) In the lecture, it was stated without proof that $e^{j\omega_0 t}$ is an eigen-signal of an LTI system $h(t)$. In other words, when the LTI system operates on an input signal $x(t) = e^{j\omega_0 t}$ [or $x(t)$ is sent into the LTI system], the output $y(t)$ is merely a scaled version of $x(t)$ for all $t \in \mathbb{R}$. Show that the scaling factor is

$$H(j\omega_0) = \int_{-\infty}^{\infty} h(t) e^{-j\omega_0 t}\, dt. \tag{1}$$

Recall that the input–output relation of an LTI system is related by $y(t) = h(t) * x(t)$.

Problem 3 (Fourier Transform) Compute Fourier transforms for the following signals. You must do the complete computation without relying on the tables.

a) $x(t) = e^{-2(t-1)}u(t-1)$

b) $x(t) = e^{-|t+2|/3}$ (Explicitly show how the absolute sign is removed.)

c) $x(t) = \text{rect}(2t+1)$ (Final result must be represented in form of a sinc function.)

d) $x(t) = 1 + t$ for $t \in [-1, 0]$ otherwise $x(t) = 0$ (Show the details of integration by parts.)

Problem 4 (Inverse Fourier Transform and Properties) Compute the inverse Fourier transform for the following signals. You must calculate the results using both i) the direct evaluation method based on the definition of the inverse FT and ii) the table of Fourier transform properties. (Try part ii after Monday's lecture.)

a) $\delta(\omega+1)+\delta(\omega-1)+j\delta(\omega+3)-j\delta(\omega-3)$

b) $\text{rect}(3\omega-2)$

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.