ECE 301 (Section 001) Homework 11
Spring 2024, Dr. Chau-Wai Wong
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Problem 1 (Bonus, 20') (Various Fourier Transform Properties)

a) Use a procedure similar to that on slide 63 of Lecture 19, prove that differentiation in frequency domain corresponds to multiplication by jt in the time domain, namely,

$$-jtx(t) \overset{\mathcal{FT}}{\longleftrightarrow} \frac{dX(j\omega)}{d\omega}.$$

b) Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure 1. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform. [Hint: The subplot (d) may need the result proved in part a).]

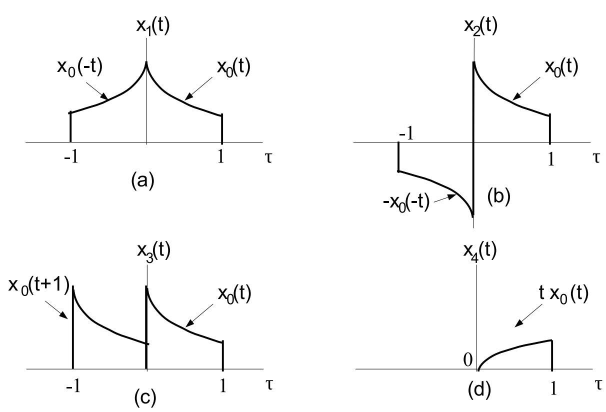

Figure 1: Signals for Fourier transform calculation.

Problem 2 (Windowed Time-Domain Signals) Find and plot the Fourier transform of the following windowed sinusoidal signals.

a)

$$x(t) = \begin{cases} \cos(10t), & -10 \le t \le 10. \\ 0, & \text{elsewhere.} \end{cases}$$

b)

$$x(t) = \begin{cases} \cos(10t), & 0 \le t \le 10. \\ 0, & \text{elsewhere.} \end{cases}$$

Hint: Use the multiplicative property of the Fourier transform.

Problem 3 (Lowpass Filter) A lowpass filter $H(\omega)$ has the frequency response shown in Figure 2.

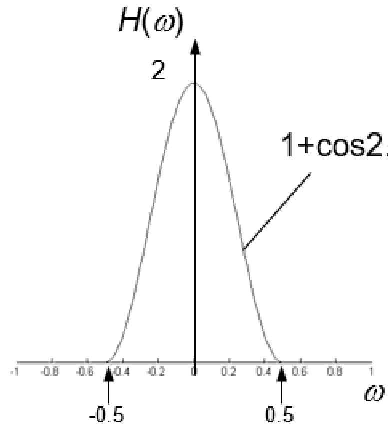

Figure 2: Frequency response of the lowpass filter $H(\omega)$.

a) Compute the impulse response $h(t)$ of the filter.

b) Compute the response $y(t)$ when the input is $x(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$, $-\infty < t < \infty$.

c) Compute the response $y(t)$ when $x(t) = \text{sinc}\left(\frac{t}{4\pi}\right)$, $-\infty < t < \infty$.

Note: You will need to use Fourier tables.

Problem 4 (DTFT and Inverse DTFT) Solve the following using the definitions *or* the properties.

a) $\mathcal{F}\{u[n-2] - u[n-6]\}$

b) $\mathcal{F}\{2^n \sin(\pi n/4) u[-n]\}$

c) $\mathcal{F}\{\sin(\pi n/2) + \cos(7\pi n/3)\}$

d) $\mathcal{F}^{-1}\left\{\frac{e^{-j\omega} - \frac{1}{3}}{1 - \frac{1}{3}e^{-j\omega}}\right\}$

Problem 5 (Bonus, 20') (Mathematical Maturity Training. This problem is difficult, so proceed with caution.) In Lecture 19, I provided a few supplemental slides discussing various sufficient conditions for Fourier transform to exist. I didn't spend time on that because, for most engineering applications, we don't encounter pathologically behaved functions; blindly invoking Fourier series/transform formulas is likely to be fine. However, for those of you who are mathematically rigorous and/or who want to pursue a graduate degree that usually requires mathematical maturity, understanding pathological examples is important. Please read Sections 3.4 and 4.1.2 of Oppenheim et al.'s book, and then use 3–5 sentences to summarize different sufficient conditions for convergence and use your own words to describe the pathological functions.

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.