ECE 301 (Section 001) Homework 12
Spring 2024, Dr. Chau-Wai Wong
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Problem 1 (Triangle, Rectangle, and Sinc Functions)

a) Using the definition of convolution, show via integration that $y(t) = H\,\mathrm{rect}(t/2W) * H\,\mathrm{rect}(t/2W)$ is a triangle. What are the width and height of the triangle?

b) Use the definition or the properties to compute the inverse DTFT of the following frequency functions $X(e^{j\Omega})$.

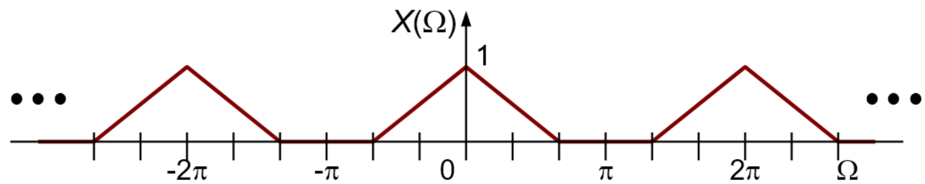

Figure 1: Frequency domain representation $X(e^{j\Omega})$ of a discrete-time signal $x[n]$.

Problem 2 (Windowing Effect and Frequency Resolution) In this problem, we will investigate the frequency resolution of Fourier transform. We investigate two neighboring musical notes, C_4 at $f_1 = 261.63$ Hz and $C_4^{\#}$ at $f_2 = 277.18$ Hz. You can play with a virtual piano here:

`https://recursivearts.com/virtual-piano/`

We make an oversimplified assumption that the audio signal of each musical note is in the form of $x_i(t) = \cos(2\pi f_i t)$, $t \in \mathbb{R}$. A causal time window $t \in [0, 2T]$ of unit gain is applied to "generate" signals of finite length. (To simplify the problem, we investigate the continuous-time signal. The analysis of the discrete-time signal is similar.)

a) With the help of the CTFT table, calculate the Fourier spectra for the windowed $x_i(t)$, for $i = 1, 2$.

b) What is the angular frequency of the first zero-crossing on the right-hand side of the magnitude spectrum of note C_4? What is the angular frequency of the first zero-crossing on the left-hand side of the magnitude spectrum of note $C_4^{\#}$?

c) What is the smallest T that can clearly separate the peaks of the two neighboring musical notes?

Problem 3 (DFT Calculations and Convolution Property)

a) Compute the DFT of the following signals by hand and plot $X[k]$. Label the low and high frequencies. Feel free to verify your results using Matlab command `fft`. For the following problems, we use vector notation to represent finite length signals. For example $\mathbf{x} = [-1, 2, 3]$ means a signal with $x[0] = -1$, $x[1] = 2$, and $x[2] = 3$.

(i) DFT of $[1, 1, -1, -1]$.

(ii) DFT of $[1, 0, 1, 0]$.

(iii) Inverse DFT of $[0, 0, 1, 0]$.

b) Show that the linear convolution result of $\mathbf{x} * \mathbf{y}$, where $\mathbf{x} = [1, 1]$ and $\mathbf{y} = [1, 1, -1, -1]$ is the same as the result of $\text{IDFT}\left\{\text{DFT}([\mathbf{x}, 0, \ldots, 0])\,\text{DFT}([\mathbf{y}, 0, \ldots, 0])\right\}$. Note that you need to determine the correct numbers of zeros padded to $\mathbf{x}$ and $\mathbf{y}$, respectively. You may use Matlab command `fft` to help you calculate DFT.

Problem 4 (10', bonus) (ClassEval) Have you completed ClassEval? It can be found at:

`http://go.ncsu.edu/cesurvey`

Grading: (a) Yes = 10 points, thank you! Please attach the screenshot of the confirmation page. (b) I promise to do it soon = 2 point for good intentions. (c) No = 0 points, a possibly honest answer, but why not spend 5 minutes and get 10 points?

Problem 5 (5', bonus) (Computational Complexity in Time Domain vs. in Frequency Domain) In class, you've seen that calculating the convolution between two time-domain sinc functions in the frequency domain can significantly lower the computational cost. Argue under what scenario time-domain calculation can significantly lower the computational cost. Give a concrete example in terms of two signals.

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.