ECE 301 (Section 001) Homework 3
Spring 2024, Dr. Chau-Wai Wong
Graduate TA in Charge: Jordan Zhang

Problem 1 (Complex Exponentials)

a) What is the signal $y(t) = Ce^{at}u(t)$ as shown in Figure 1? That is, use your new understanding of complex exponential signals to determine A, θ, r, and ω such that $C = Ae^{j\theta}$ and $a = r + j\omega$. Note from the plots that $\text{Re}\{y(0)\} = 0$ and $\text{Im}\{y(0)\} = 2$.

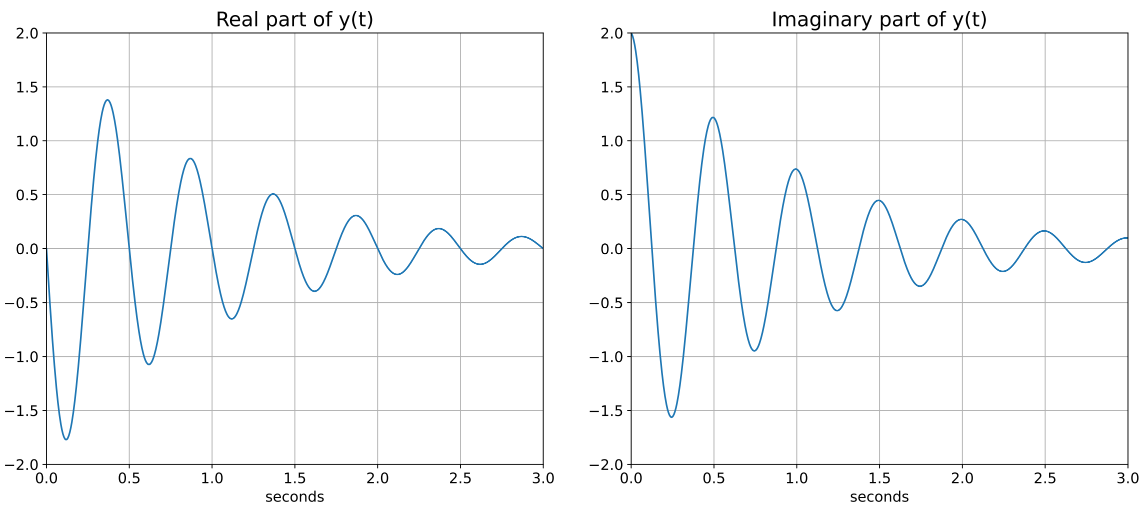


Figure 1: Continuous-time complex exponential, $y(t)$.

b) Determine the discrete time signal $y[n] = C\alpha^n u[n]$ shown in Figure 2. That is, determine A, θ, R, and ω_0 such that $C = Ae^{j\theta}$ and $\alpha = Re^{j\omega_0}$. You can assume $\text{Re}\{y[0]\} = \text{Im}\{y[0]\} = 1/\sqrt{2}$. (Hint: The trigonometric identity, $\cos^2\phi + \sin^2\phi = 1$, may be helpful.)

Problem 2 (The Periodicity of a Discrete-Time Signal)

a) Determine whether or not the following signals are periodic. If periodic, determine its fundamental period, otherwise explain why it is aperiodic.

i) $x[n] = \cos(n/6 + \pi/4)$

ii) $x[n] = e^{j\frac{3\pi}{2}n} + e^{j\frac{5\pi}{3}n}$

b) Use Matlab to plot the real parts of the above signals to verify your analytic results. Append your code and plots in your submission. Add whitespace when necessary to enhance the readability of your code. See Google's Style Guide.

c) (5', bonus) Consider the periodic discrete-time exponential time signal

$$x[n] = e^{jm(2\pi/N)n}.$$

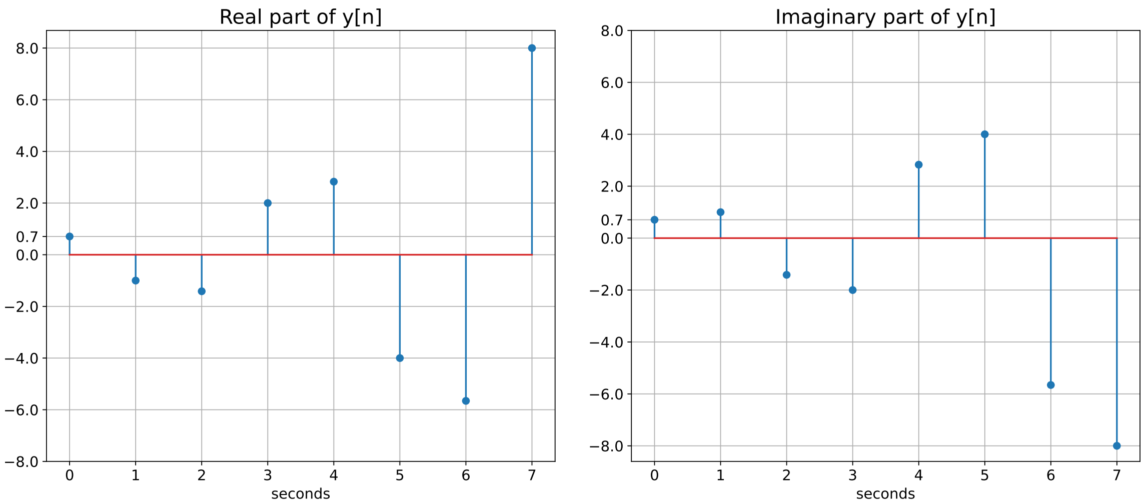


Figure 2: Discrete-time complex exponential, $y[n]$.

Show that the fundamental period of this signal is

$$N_0 = N/\gcd(m, N),$$

where $\gcd(m, N)$ is the greatest common divisor of m and N—that is, the largest integer that divides both m and N an integral number of times. For example,

$$\gcd(2, 3) = 1, \quad \gcd(2, 4) = 2, \quad \gcd(8, 12) = 4.$$

Note that $N_0 = N$ if m and N have no factors in common.

Problem 3 (Basic Operations on Signals)

a) A discrete-time signal $x[n]$ is shown in Figure 3(a). Sketch carefully each of the following signals by hand:

i) $x[2n]$

ii) $x\left[\frac{n}{2}\right]$

iii) $x[n]\left(u[n] + \delta[n-2]\right)$

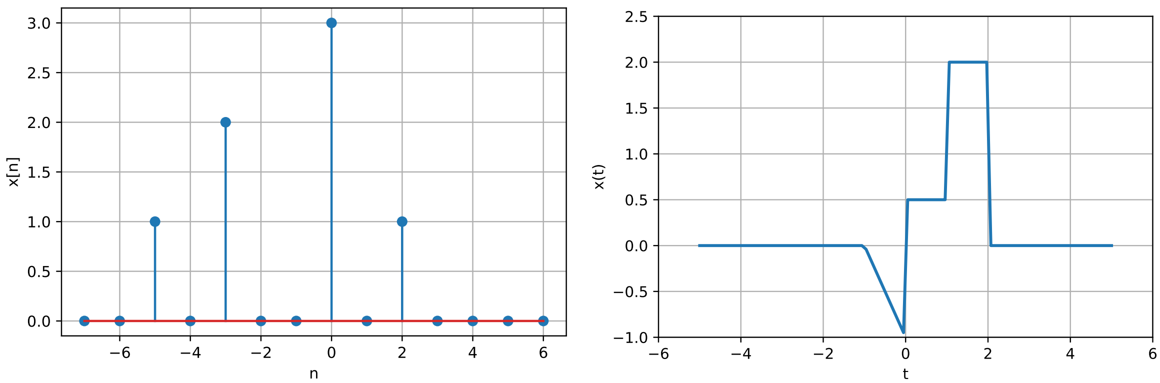
b) A continuous signal $x(t)$ is shown in Figure 3(b). Sketch carefully each of the following signals by hand:

i) $x(t-3)$

ii) $x(1-t/2)$

iii) $x(t)\left[u(-t) + \delta(t-4)\right]$

Problem 4 (Sifting Property of the Impulse Functions)



(a) (b)

Figure 3: (a) Discrete-time signal, $x[n]$. (b) Continuous-time signal, $x(t)$.

a) Simplify the following expressions:

i) $\left(\frac{\sin(n+\pi)+8}{n^3-2}\right)\delta[n]$

ii) $\left(\frac{\cos(n\pi+\pi)}{-(n+1)^2}\right)\delta[n-3]$

iii) $\left(\frac{\cos t}{t^{64}-4}\right)\delta(t)$

iv) $\left(\frac{\sin(2k\omega)}{\omega}\right)\delta(\omega)$

b) Evaluate the following integrals and sums:

i) $\sum_{n=-\infty}^{\infty}\delta[n-6]\gamma^{n-3}\cos\left(\frac{\pi}{12}(6+n)\right)$

ii) $\int_{-\infty}^{\infty}\delta(t-4)\sin(\pi t)dt$

iii) $\int_{-\infty}^{\infty}\delta(3-t)f(1+t^2)dt$

iv) $\int_{-\infty}^{\infty}\delta(x-2)e^{(x-1)}\cos\left(\frac{\pi}{2}(x^2-5x+4)\right)dx$

Problem 5 (Bonus, 10 pts) (The Fundamental Period of the Sum of Two Signals)

a) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic, and what is the fundamental period of $x(t)$ if it is periodic? (Hint: Start by using the definition of periodic function.)

b) Let $x_1[n]$ and $x_2[n]$ be periodic sequences with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] = x_1[n] + x_2[n]$ periodic, and what is the fundamental period of $x[n]$ if it is periodic?

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.