ECE 301 (Section 001) Homework 4
Spring 2024, Dr. Chau-Wai Wong
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Problem 1 (Impulse Function and Step Function) (20 pts)

a) Find and sketch the first derivatives of the following signals:

i) $x(t) = u(t) - u(t-a)$, $a > 0$,

ii) $x(t) = t[u(t) - u(t-a)]$, $a > 0$.

iii) $x(t) = \text{sign}(t) = \begin{cases} 1, & t \geq 0, \\ -1, & t < 0. \end{cases}$

b) Given that the $\delta(\cdot)$ is the unit impulse function, prove the following property:

$$\int_{-\infty}^{\infty} \delta(ax)dx = \int_{-\infty}^{\infty} \frac{\delta(t)}{|a|}dt = \frac{1}{|a|},$$

where a is a scalar. Can you claim that $\delta(ax) = \frac{1}{|a|}\delta(t)$? If yes, in what sense? If no, why.

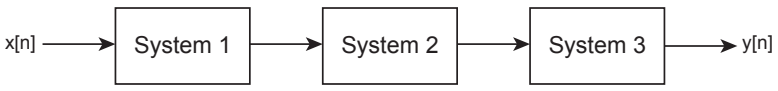
Problem 2 (Interconnected Systems) (20 pts) Consider three systems with these input–output relationships:

$$\text{System 1 (upsampling):} \quad w[n] = \begin{cases} x[n/3], & n \text{ multiple of 3}, \\ 0, & \text{otherwise}. \end{cases}$$

$$\text{System 2 (smoothing):} \quad z[n] = w[n] + 2w[n-1] + w[n-2].$$

$$\text{System 3 (downsampling):} \quad y[n] = z[3n].$$

Suppose that these systems are connected in series, as shown below.



i) Find the input–output relationship, namely, write $y[n]$ as a function of $x[n]$, for the overall interconnected system.

ii) Is it time invariant? Is this system linear? (This subquestion should be answered without using the derivation steps that will be covered in Monday, 1/30's lecture.)

(Hint: You can draw plots to see "physically," how a three-sample input signal, e.g., $x[0] = 1$, $x[1] = -1$, $x[2] = 2$, $x[n] = 0$ for all other n, evolves as it passes each block of the system.)

Problem 3 (Properties of Discrete Systems) (20 pts) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y[n]$ denotes the system output and $x[n]$ denotes the system input.

a) $y[n] = x[n]\,x[n-1]\,x[n+1]$

b) $y[n] = \begin{cases} x[n-1], & n \leq 0, \\ x[n+1], & n > 0. \end{cases}$

c) $y[n] = \cos(x[n])$

Problem 4 (Properties of Continuous Systems) (20 pts) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y(t)$ denotes the system output and $x(t)$ is the system input.

a) $y(t) = (t-2)\,x(t+2)$

b) $y(t) = x(t-1) + x(4-t)$

c) $y(t) = x\big(\sin(t)\big)$

d) (Bonus, 7') $y(t) = \begin{cases} 0, & x(t) < 1, \\ \int_0^1 x(t-\tau)d\tau, & x(t) \geq 1. \end{cases}$

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.