

ECE 301 (Section 001) Homework 8
Spring 2024, Dr. Chau-Wai Wong
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Problem 1 (Deep Learning with Matlab) In recent updates, Matlab has put together well-guided tutorials for deep learning. This is one set of tutorials on deep-learning-based “Image Data Workflows”. Complete the following tutorials by running the example code:

“Classify Webcam Images Using Deep Learning.”

Write a concise report consisting of key source code, images, and your explanations. These tutorials may give you ideas about your projects in other courses.

Problem 2 (Projection and PCA) You are given a data matrix

$$\mathbf{X} = \begin{bmatrix} -0.92 & 1.09 & -1.35 & 2.06 & -0.60 & -0.28 \\ -0.08 & 1.15 & -1.67 & 1.08 & -1.14 & 0.66 \end{bmatrix}.$$

and its principal component $\mathbf{u} = [0.7474, 0.6644]^T$.

- a) Plot the data points using circles and the principal component using a line segment with one end at $[0, 0]^T$ on a 2D plane using Matlab.
- b) Calculate the projection c_i for each data point \mathbf{x}_i to \mathbf{u} . Note that c_i can be negative.
- c) Calculate the sample variance of $\{c_i\}_{i=1}^n$, namely,

$$\widehat{\text{Var}}(\{c_i\}) = \frac{1}{n-1} \sum_{i=1}^n (c_i - \bar{c})^2,$$

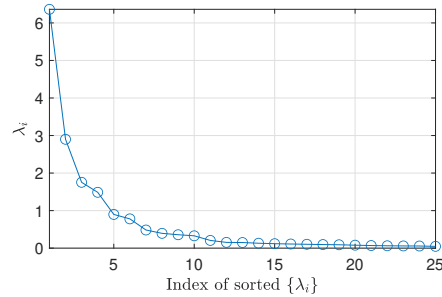
where $\bar{c} \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^n c_i$ is the sample mean.

- d) Repeat b) and c) using another unit vector $\mathbf{u}_a = [0.8, 0.6]^T$. Is the newer sample variance smaller than that in b)? Can you explain why?

Problem 3 (PCA on Downsampled Yale Face Database) In this problem, we will explore PCA as a visualization tool for Yale Face Database. Download the .m files and the database. Extract the face image files into a folder named `yalefaces` and put the .m files at the same level of the folder. Call this folder “`problem_pca_face/`”. Open Matlab, change your “Current Folder” to “`problem_pca_face/`”, and open `main_pca_visualization.m`.

- a) Run the code corresponding to this part only, describe the data structure of variable `img_buffer`. Set `preview_img_flag` to 1, re-run the code to visually inspect the whole database.

- b) Complete Matlab function `[V, Lambda_mat] = PcaViaKlt(data)` by implementing PCA using eigendecomposition on a sample covariance matrix of the face data. The detailed information about the input and outputs are given in the comments of the incomplete function. You may use built-in function `eig` for eigendecomposition. If your implementation is correct, after running the code of b), you will obtain a plot similar to the following.



- c) Run the code of c) to visualize a few dominating eigenvectors. Comment on whether they reflect some characteristics of the faces you saw in a).
- d) The code of d) projects each face image (coming from one of the four selected classes) onto a 2D space. Comment on PCA’s data visualization performance in this specific example.

Problem 4 (Orthogonal Projection) (Please attempt this problem after Monday’s lecture.) Consider the set of inconsistent linear equations $\mathbf{Ax} = \mathbf{b}$ given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

- a) Find the least-squares solution to these equations.
- b) Find the “hat” matrix \mathbf{H} . Using Matlab, numerically verify $\mathbf{H} = \mathbf{H}^2$. Argue why.
- c) Find the best approximation $\hat{\mathbf{b}} = \mathbf{H}\mathbf{b}$ to \mathbf{b} . Find the vector $\mathbf{b}' = (\mathbf{I} - \mathbf{H})\mathbf{b}$ and show numerically that it is orthogonal to $\hat{\mathbf{b}}$.
- d) What does the matrix $\mathbf{I} - \mathbf{H}$ represent? If \mathbf{H} is called the “orthogonal projector,” can you think of a name for $\mathbf{I} - \mathbf{H}$? Numerically verify $\mathbf{I} - \mathbf{H} = (\mathbf{I} - \mathbf{H})^2$. Argue why.
- e) In a 3-dimensional coordinate system, draw the column vectors of matrix \mathbf{A} , the column vector space of \mathbf{A} , \mathbf{b} , $\hat{\mathbf{b}}$, and \mathbf{b}' . Make sure that the drawing is reasonably accurate which can reflect the relationship among these quantities.

Problem 5 (Mid-Semester Class Evaluation, Bonus 10’) To get the bonus points, please take a screenshot of the confirmation page after submitting the survey and use the screenshot as the answer to this problem. Survey link:

<https://forms.gle/XTtHrzn6nezVZhEH7>

Group Study (1', bonus) In-Person: Take a selfie with all group members' faces in the photo. Capture in the photo the homework assignment sheet that you are working on. Zoom: Take a screenshot of the whole team with everyone's webcam capturing his/her face. One of you will share the screen showing the specific homework assignment sheet.

Include the screenshot/selfie in your own homework submission as the last "problem." Your screenshot/selfie gets you 1 bonus point; your group members need to do it separately to earn their bonus points.