ECE 301 (001) Linear Systems
Spring 2024 Midterm Exam 1
Instructor: Dr. Chau-Wai Wong

This is a closed-book, closed-lecture notes exam. One-sided, letter-sized, handwritten cheat sheet is allowed. Calculators are not allowed. You are required to answer ***four complete problems*** of your choice. (If you provide answers to all five problems, points will be assigned only for four randomly chosen problems.) Submission instructions for in-class students:

1. Once the instructor says "stop writing," please use your mobile phone to "scan" every answer sheet. (After scanning, you must turn in the physical copy of your answer sheets to the instructor, or you will receive zero for this exam.)

2. Do not forget to scan those pages on the back if you wrote answers on them. Double check the scanned pages one by one on your phone to make sure they are properly captured.

3. You have a time window of 6 hours after the exam to upload your answers to Gradescope. Upload your scanned pages as a PDF file to Gradescope and tag each page.

4. There will be a 5-point penalty for every late hour after the submission deadline.

5. Turn in the physical copy of your answer sheets to the instructor in the front of the classroom.

If you take the exam at a university testing center, the proctoring staff will scan your papers and send them to Dr. Wong. Dr. Wong will later forward the scanned PDF to you, and you will be responsible for uploading the PDF file to Gradescope and tagging each page within 24 hours of receiving Dr. Wong's email.

Problem 1 (25 pts) [Geometric Series] Let

$$S = \sum_{m=a}^{-1} r^m, \tag{1}$$

where $r \in \mathbb{R}$, $a \in \mathbb{Z}$, and $a < -1$. Derive the analytic form of S without the help of the geometric series formula. Show intermediate steps clearly. Remember to deal with different cases separately.

Problem 2 (25 pts) [Operations on Signals]

(a) You are given a discrete-time signal $x[n] = \delta[n+1] - 2\delta[n] + 5\delta[n+3] - \delta[n-6]$, $n \in \mathbb{Z}$. Sketch carefully the following signal.

$$y[n] = x[2n-1]. \tag{2}$$

Partial points will be given for intermediate calculations/manipulations you write down.

(b) You are given signal $x(t) = 2\cos(t)\,[u(t+\pi/2) - u(t-\pi/2)]$, $t \in \mathbb{R}$.

(i) Sketch $x(t)$. Clearly provide important tick values on the t-axis and x-axis to facilitate a precise graphical description of $x(t)$.

(ii) Calculate the first derivative of $x(t)$. Note that $(uv)' = u'v + uv'$, $(\sin x)' = \cos x$, and $(\cos x)' = -\sin x$.

(iii) Sketch the first derivative of $x(t)$ using (I) the result of (ii), or (II) your observation from the graph you sketched for (i). If you proceed via route (II), justify in words how you obtained the various components in your plot.

Problem 3 (25 pts) [Periodicity and Sifting Property]

(a) Determine whether or not the following signals are periodic. If periodic, determine its fundamental period, otherwise explain why it is aperiodic. (If you use off-the-shelf formulas, make sure they are applied correctly. You may also try to find the period by starting from the definition, which allows us to give you partial credits even if the final result is wrong.)

(i) $x(t) = \cos(19t/13 + \pi/7)$.

(ii) $x[n] = \cos\left(\frac{2025}{2024}\pi n\right) + \exp\left(j\frac{17}{2}\pi n\right)$.

(b) Simplify/evaluate the following expressions. To save your time, there is no need to copy the following equations to your answer sheet.

(i) $\sum_{n=-\infty}^{\infty} (n^3 + n^2 + 1)\delta[n+1]$.

(ii) $\int_{-\infty}^{\infty} \delta(x-1)e^{x-1}\cos\left(-\frac{\pi}{32}(x^2 - 5x + 4)\right) dx$.

Problem 4 (25 pts) [Convolution] You are given

$$x(t) = e^{4t}u(t-3), \tag{3a}$$

$$h(t) = e^{-3}[u(t+2) - u(t+1)], \tag{3b}$$

for $t \in \mathbb{R}$. You are guided to calculate $y(t) = x(t) * h(t)$ as follows:

(i) Use either a graphical or analytic approach to determine all subintervals on which $y(t)$ will have different expressions.

(ii) Evaluate $y(t)$ on each subinterval.

Show intermediate steps to gain full points.

Problem 5 (25 pts) [System Properties] For the given system

$$y(t) = x\big(\sin(t)\big), \tag{4}$$

determine each of the following properties:

(i) memoryless,

(ii) causal,

(iii) invertible,

(iv) BIBO stable,

(v) time invariant, and

(vi) linear.

You must justify with mathematical expressions and/or in words at each intermediate step.

Answer sheet for Problem [ ] Name: Obtained Points:

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Draft paper

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