ECE 301 (Section 001) Bonus Problems
Spring 2025, Dr. Chau-Wai Wong

Problem 1 (20', bonus) (Forward z-transform) Determine the z-transform for each of the following sequences. Sketch the pole–zero plot and indicate the region of convergence. Indicate whether or not the discrete-time Fourier transform of the sequence exists.

a) $x_1[n] = \left(\frac{1}{2}\right)^{n+1} u[n+3]$.

b) $x_2[n] = \left(-\frac{1}{3}\right)^{n} u[-n-2]$.

c) $x_3[n] = \left(\frac{1}{4}\right)^{n} u[3-n]$

Problem 2 (20', bonus) (Inverse z-transform) Consider the discrete-time system described by

$$H(z) = \frac{1}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}.$$

a) Find the impulse response $h[n]$ under the following conditions and comment on whether it is a stable and/or causal system for each of the following cases:

(i) If the ROC is $|z| > 1$.

(ii) If the ROC is $|z| < \frac{1}{2}$.

(iii) If the ROC is $1 > |z| > \frac{1}{2}$.

b) Sketch the magnitude of the frequency response of this system $|H(e^{j\omega})|$ using the pole–zero plot. Note that this is unaffected by the ROC.

Problem 3 (20', bonus) (Discrete-Time Frequency Response) A causal, discrete-time LTI system has a transfer function

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$

a) Write the difference equation corresponding to $H(z)$.

b) Plot the pole–zero diagram of $H(z)$ and indicate the region of convergence for this system.

c) Using the pole–zero plot, sketch an estimate of $|H(e^{j\omega})|$.

d) State whether the following are true or false, along with a reason:

(i) The system is stable.

(ii) The impulse response approaches a (finite) constant for large n.

(iii) The magnitude of the frequency response has a peak at approximately $\omega = \pm\pi/4$.

(iv) The system has a stable and causal inverse.