ECE 792-41 Homework 2

Material Covered: Linear Regression Model, Geometric Interpretation; Auto-Regressive (AR), Moving-Average (MA) and ARMA Processes, Yule-Walker Equations, Autocorrelation Functions.

Problem 1 Scheffe 1.1 (For ω, you need to do partial differentiation; for Ω, skip the derivation and directly write out the estimate for $[\alpha\ \beta\ \gamma]^T$ using the normal equation in matrix-vector form. Do not simplify.)

Problem 2 Scheffe 1.2

Problem 3 Scheffe 1.4

Problem 4 A first-order autoregressive (AR) process $\{u(n)\}$ that is real-valued satisfies the real-valued difference equation

$$u(n) + a_1 u(n-1) = v(n)$$

where a_1 is a constant and $\{v(n)\}$ is a white-noise process of variance σ_v^2. Such a process is also referred to as a *first-order Markov process*.

(a) Suppose in practical implementation, the generation of the process $\{u(n)\}$ starts at $n = 1$ with initialization $u(0) = 0$. Determine the mean of the actual $\{u(n)\}$ process that we have obtained. Under what conditions $E[u(n)]$ converges to a constant and what the constant is?

(b) Now consider the case when $\{v(n)\}$ has zero mean. Determine the variance of the actual $\{u(n)\}$ process that we have obtained. Under what conditions $\mathrm{Var}[u(n)]$ converges to a constant and what the constant is?

(c) For the conditions specified in part (b), find the autocorrelation function of the AR process $\{u(n)\}$. Sketch this autocorrelation function when $n \gg k$, for the two cases $0 < a_1 < 1$ and $-1 < a_1 < 0$.

Problem 5 Consider an autoregressive process $\{u(n)\}$ of order 2, described by the difference equation

$$u(n) = u(n-1) - 0.5\,u(n-2) + v(n)$$

where $\{v(n)\}$ is a white-noise process of zero mean and variance 0.5.

(a) Write the Yule-Walker equations for the process.

(b) Solve these two equations for the autocorrelation function values $r(1)$ and $r(2)$.

(c) Find the variance of $\{u(n)\}$.

Problem 6 Consider an MA process $\{x(n)\}$ of order 2 described by the difference equation

$$x(n) = v(n) + 0.75\,v(n-1) + 0.25\,v(n-2)$$

where $\{v(n)\}$ is a zero mean white noise process of unit variance. The requirement is to approximate the process by an AR process $\{u(n)\}$ of order M. Do this approximation for the orders $M = 2$ and $M = 5$, respectively.