ECE 792-41 Statistical Methods for Signal Analytics

Project 2: Spectrum/Frequency Estimation & Adaptive Filtering

This is project to help you review and practice the spectrum estimation theory and techniques learned. The project should be completed *individually*. You are expected to submit a **concise write-up** consisting of results and discussions of your findings; include appropriate plots as applicable. No need to include detailed background reviews that mainly repeat the techniques taught in lectures, unless you are using any approaches that are not covered in lectures.

You are allowed to use the MATLAB built-in functions for auxiliary purposes, such as plotting, FFT, numerical optimization, etc. You may use the linear prediction and related codes from your previous project.

You should sign the **Honor Pledge** at the beginning of the report: "*I pledge in my honor that I have not given or received any unauthorized assistance on this report*".

1. Spectrum Estimation

1.1. Write functions to generate five random processes. Each of them is generated using some model as explained below and then corrupted by white noise for about 25dB of signal-to-noise ratio (SNR). The first process uses an autoregressive (AR) model; the second process uses a moving average (MA) model; the third uses an ARMA model; the fourth contains several sinusoids at different frequencies; and the fifth contains the sum of two sinusoids and an AR process. The input argument of your function specifies the SNR as well as the order and parameters of the processes to be generated. Derive the expression of the Power Spectrum Density (P.S.D.) for each process, and use computer to plot this "true" P.S.D.

1.2. Apply the parametric spectrum estimation methods and non-parametric methods (periodogram, averaged periodogram, and MVSE) from the lectures, and examine the estimated spectrum by each method. You can choose a few reasonable sets of parameters for each type of process (e.g., AR/MA of order 3 to 10). Consider the cases of having a short record (say, 32 to 64 sample points) versus having a long record (say, a thousand sample points) available to you to do the spectrum estimation. Compare the results by different methods and also with the true P.S.D. How good is each estimation approach? Under what situation does it give good or bad estimation? Discuss your findings.

2. Frequency Estimation

2.1. Generate 9 groups of 8-second signals with SNR = −40, −30, …, 0, …, 30, 40 dB, respectively, with sampling frequency at 150 Hz. Each group of signals consists of 1,000 realizations of real-valued sinusoid with *f* = 60 Hz and random phase buried in the white Gaussian noise.

(a) Implement the following spectrum/frequency estimation methods, periodogram, averaged periodogram, MVSE, MUSIC, and IAA [1–2].

(b) Write a peaking searching algorithm capable of outputting from the estimated spectrums generated in (a) the index of the peak, i, and its peak value $S(i)$, as well as the value of the left and right neighbors of the peak, $S(i-1)$ and $S(i+1)$. Derive a deterministic function that takes as input $\{i, S(i-1), S(i), S(i+1)\}$ and output an adjusted index of peak $i^* \in (i-1, i+1)$ that is the peak position of a quadratic curve passing through $S(i-1)$, $S(i)$, and $S(i+1)$. Use this deterministic function to improve the accuracy of all the algorithms in (a).

(c) Examine how the change of parameter values such as the length of window, the number of segments, the size of correlation matrix affects the mean squared error (MSE) between the frequency estimates and the ground truth. Is the effect of the change of parameter values the same at different SNR levels? Discuss your findings.

2.2. Generate one-hour long sinusoidal signals of time-varying frequency with frequency changing every T seconds. Assume that the time varying frequency has a nominal value of 60 Hz and its fluctuation follows an AR(1) process with $a_1 = -0.9$ and $\sigma_e = 0.05$ Hz. Assume the sampling frequency of the sinusoidal signal is f_s = 150 Hz, generate separate signals for T = 0.5, 1, 2, 4, 8 seconds. Answer the following questions.

(a) Using the equation for frequency modulation (FM) in the integral form, prove that in the discrete case

$$x(n) = \cos[\phi_0 + 2\pi T_s \sum_{\ell=1}^{n} f(\ell)],$$

where ϕ_0 is a random initial phase, T_s is the sampling period, and $f(\ell)$ is the averaged frequency during ℓth sampling period.

(b) Use the frequency estimation methods in 2.1 to estimate the instantaneous dominating frequency of the generated sinusoidal signals. Plot your results in terms of MSE as a function of T. What do you observe from your results? Discuss your findings.

(c) Now add white Gaussian noise at a SNR = −40, −30, …, 0, …, 30, 40 dB to your sinusoidal signal and repeat part (a). What is the effect of noise levels on the frequency estimates? Which method gives you the best frequency estimates? Why?

3. Frequency Tracking on Heartbeat Signal

3.1. Below is a concise introduction to heart rate estimation using face videos appeared in [3]: “Contact-free monitoring of the heart rate using videos of human faces is a user-friendly approach compared to conventional contact based ones such as electrodes, chest belts, and finger clips. Such monitoring system extracts from a face video a 1-D sinusoid-like face color signal that has the same frequency as the heartbeat. The ability to measure heart rate without touch-based sensors is attractive and gives it potentials in such applications as smart health and sports medicine.” You are given a preprocessed face color signal sampled at 30 Hz in which the heart rate fluctuates between 78 and 85 beats per minute (bpm). When necessary, use the quadratic interpolation function you wrote in 2.1(b) to improve the displayed frequency resolution.

(a) Use the periodogram with 98% window overlap to estimate the heart rate signal. Decide an optimal window length between 1 to 10 seconds.

(b) Try other frequency estimation methods and plot the estimated signals in one plot. Compare the results from various frequency estimation methods.

(c) Consider the face color signal to be an AR signal that can be modeled by a time-varying all-pole filter. Use the LMS filter to track the AR coefficients of the all-pole filter and output the estimated heart rate signal. Discuss the results. (Hint: You may want to first bandpass filter the face color signal to reduce the order of the AR model that tracks reasonably well.)

[1] T. Yardibi, J. Li, P. Stoica, M. Xue, and A. B. Baggeroer, “Source localization and sensing: A nonparametric iterative adaptive approach based on weighted least squares,” IEEE Transactions on Aerospace and Electronic Systems, vol. 46, no. 1, 2010.

[2] O. Ojowu, J. Karlsson, J. Li, and Y. Liu, “ENF extraction from digital recordings using adaptive techniques and frequency tracking,” IEEE Transactions on Information Forensics and Security, vol. 7, no. 4, pp. 1330–1338, 2012.

[3] Q. Zhu, C.-W. Wong, C.-H. Fu, and M. Wu, “Fitness heart rate measurement using face videos,” IEEE International Conference on Image Processing (ICIP’17), Beijing, China, 17–20 Sep. 2017.